Connectivity Properties of Random Waypoint Mobility Model for Ad Hoc Networks

Pasi Lassila Esa Hyytiä Henri Koskinen
Networking Laboratory, Helsinki University of Technology,
P.O. Box 3000, FIN 02015 TKK, Finland

Email: {Pasi.Lassila, Esa.Hyytia, Henri.Koskinen}@tkk.fi

***Abstract*— We study the connectivity properties of an ad hoc network consisting of a given number of nodes each moving according to the Random Waypoint mobility model. Connectivity properties of networks with uniformly distributed nodes are well known, but the movement in the RWP model results in a spatial node distribution which is not uniform. Approximations are provided for the probability that the network is connected, as well as for the mean durations of the connectivity periods. The accuracy of the approximations is compared against extensive numerical simulations. Especially, for the probability of connectivity, based on new results on the exact node location distribution, an approximation is given that is remarkably accurate. Furthermore, it is shown by numerical examples that in sparse network the mobility has a positive effect on connectivity, whereas in dense network the situation becomes the opposite. For the mean length of the connectivity periods an approximation is derived that gives accurate results in the important region where the probability of connectivity rises rapidly.**

***Index Terms*— ad hoc networks, mobility modelling, k-connectivity, RWP.**

I. Introduction

An intrinsic property of wireless ad hoc networks is that data transmission between two nodes occurs over a multihop path. The functionality of the network critically depends on the connectivity properties of the network. These typically depend on the transmission range of the nodes, the number of nodes and the distribution of the nodes, e.g., resulting from the assumed mobility model. A common assumption for the distribution is the uniform distribution. Introducing mobility may result in a distribution that is far from uniform. Hence, understanding the impact of mobility on connectivity is essential. In this paper, we study both the probability that the network is connected and also the lengths of the connectivity (and disconnectivity) periods under the assumption of nodes moving according to the so called Random Waypoint (RWP) mobility model.

The RWP model is one of the most popular mobility models used in performance studies of ad hoc networks. Recently, several papers have appeared analyzing various properties of RWP [1], [2], [3], [4], [5]. From our point of view, one important result concerns the stationary distribution of the location of a node moving according to RWP. Approximate results for various shapes (circle, rectangle) of the RWP movement area have been obtained in [1], [3]. However, as part of our earlier work, in [4] we have derived an exact expression for the node location distribution which applies in any convex movement area (a common assumption in RWP models). The exact result has an especially appealing form in the case of a circular movement area. Additional related results on the mean arrival rate of nodes into a given subset of the RWP movement area have been given in [5].

Also connectivity in wireless multihop networks has been studied intensively. Connectivity problem deals with determining if it is possible to transfer information between any two nodes ignoring all capacity and traffic related phenomena, most notably interference effects. The most popular network model – and the one used in this study – defining when two nodes are directly connected has been the Boolean one, in which two nodes are connected if they are both within each others' transmission ranges. When the Boolean model is augmented with the common assumption that all nodes have an equal transmission range, the connectivity problem reduces to determining the distribution of the threshold range for connectivity: for a given set of nodes, this is equal to the greatest edge length in the minimum spanning tree of the nodes [6]. It has been shown in [7] that for uniformly distributed nodes in the unit square, as the number of nodes tends to infinity, the threshold range for connectivity has asymptotically the same, previously known, distribution as the threshold range for minimum degree 1, i.e., the greatest edge length in the nearest-neighbor graph. The result has been generalized to k-connectivity in [8]. Furthermore, the identity in the case $k = 1$ has been shown to hold for normally distributed points in [9]. Recently, the asymptotic distributions of the threshold range for k-connectivity when $k > 1$, for uniformly distributed points in circular and square-shaped domains have been derived in [10]. The distribution of the threshold range for k-connectivity is not known when the number of nodes is finite. The results above motivate predicting k-connectivity of finite networks by minimum degree k, as has been done, e.g., in [11]; this is also the basis of our approach.

In this paper we present approximations for the probability that a given ad hoc network with n nodes is k-connected. The nodes are assumed to move according to RWP, which concentrates more probability mass in the center of the area than to areas near the borders. In the RWP model, nodes move independently and the number of neighbors a given node has is binomially distributed with a certain parameter p. These are needed in our approximation for the probability that all nodes have at least k neighbors, which is used to approximate the probability of k-connectivity. In our first approximation, using our earlier exact results on the distribution of the node location, the parameter p can be computed exactly. Additionally, two other numerically simpler approximation schemes are given, which are based on making some additional poissonian assumptions. Our approach is similar to the one in [11], with the distinction that in [11] the binomial distribution characterizing the number of neighbors a given node has is approximated by a Poisson distribution with appropriately computed mean. Also, we have an exact result for the node location distribution, whereas in [11] an approximation has been used (although a rather accurate one). The quality of the approximations for 1-, 2- and 3-connectivity are evaluated by means of numerical simulations in a unit disk, while the approach itself is not limited to any special geometry. In the simulations, the threshold ranges for k-connectivity have been determined using the efficient algorithms given in [12]. The results show that especially our first approximation gives remarkably accurate results. We also provide an approximation to estimate the mean time that a network with n nodes is 1-connected (or disconnected). The approximation utilizes results on the arrival rate of the RWP process in a given subset of the movement area. These combined with our approximation for the probability of 1-connectivity yield an approximation for the mean connectivity periods. The approximation is validated by means of numerical simulations, and the results show that the approximation gives reasonably accurate estimates in the most important region where the probability of connectivity rises rapidly.

The paper is organized as follows. Section 2 provides the necessary theoretical background. Our approximations are derived in Section 3. Numerical results are in Section 4, and Section 5 contains the conclusions

II. Preliminaries

A. Connectivity

First some terminology and elementary definitions are introduced. We limit ourselves to undirected graphs $\mathcal{G} = (V, E)$, where V is a set of vertices (or nodes) and E is a set of edges (or links). Nodes v_i and v_j are said to be neighbors if there is an edge (v_i, v_j) in E. A path in $\mathcal{G}$ is a sequence of vertices $v_1, v_2, \ldots, v_n$ such that an edge exists in E for each (v_i, v_{i+1}), $i = 1, \ldots, n-1$. A graph $\mathcal{G} = (V, E)$ is said to be (1-)connected if for each (s, d)-pair a path exists from s to d. Also, a graph $\mathcal{G} = (V, E)$ is said to be k-connected if for each (s, d)-pair at least k node disjoint paths exist. In other words, (1-)connectivity tells us whether we can send data to each possible destination. In case of k-connectivity, each destination can be reached even if any $k-1$ nodes fail.

In (ad hoc) networks two nodes are neighbors, i.e., have an edge between them in the connectivity graph $\mathcal{G}$, iff they both can hear each other's transmissions. In a general case some links may be unidirectional, i.e., node A can hear node B transmissions but not vice versa. For simplicity, in this work it is assumed that all links are bidirectional. In particular, we assume that two nodes can hear each other if the distance between them is less than d.

B. Random Waypoint Model

In the RWP model, nodes move in a convex subset denoted by $\mathcal{A}$. In particular, each node moves independently of the others directly towards its next waypoint at a certain velocity v. Once the node reaches the waypoint, the next waypoint is drawn randomly from the uniform distribution over $\mathcal{A}$. Similarly, the velocity for the next leg is drawn independently from the velocity distribution. Furthermore, it is possible to introduce "thinking times" upon reaching the waypoints.

In the following we state the necessary results from [4] and [5] for our purposes. Let $\bar{\ell}$ denote the mean length of a leg and A the area of the domain $\mathcal{A}$. Also, let $a_1 = a_1(\mathbf{r}, \phi)$ denote the distance from point $\mathbf{r} \in \mathcal{A}$ to the border of $\mathcal{A}$ in direction ϕ and similarly, let a_2 denote the distance to the border in the opposite direction, i.e., $a_2(\mathbf{r}, \phi) = a_1(\mathbf{r}, \phi + \pi)$. Define[1]

$$h(\mathbf{r}, \phi) = \frac{1}{2} \cdot a_1 a_2 (a_1 + a_2).$$

The stationary distribution of an RWP node is given by (see [4])

$$f(\mathbf{r}) = \frac{1}{C} \int_0^{2\pi} h(\mathbf{r}, \phi)\, d\phi = \frac{h(\mathbf{r})}{C}, \tag{1}$$

where normalization constant $C = \bar{\ell} A^2$. Consequently, by normalization we get

$$\bar{\ell} = \frac{1}{A^2} \int_{\mathcal{A}} h(\mathbf{r})\, dA.$$

The mean arrival rate into a subset $\mathcal{A}_j \subset \mathcal{A}$ is given by [5]

$$\lambda(\mathcal{A}_j) = \int_{\partial \mathcal{A}_j} \lambda(\mathbf{r}, \theta(d\mathbf{r}))\, dr, \tag{2}$$

where $\theta(d\mathbf{r})$ is the direction of the tangent at point $\mathbf{r}$, and

$$\lambda(\mathbf{r}, \theta) = \frac{1}{C \cdot \mathrm{E}[1/v]} \int_0^{\pi} \sin\phi \cdot h(\mathbf{r}, \theta + \phi)\, d\phi.$$

C. Random Waypoint in Unit Disk

For unit disk the pdf of node location, denoted by $f(r)$, is given by

$$f(r) = \frac{2(1-r^2)}{C} \int_0^{\pi} \sqrt{1 - r^2 \cos\phi}\, d\phi, \tag{3}$$

where $C = 128\pi/45 \approx 8.936$ [4]. Let $\lambda(r, d)$ denote the mean arrival rate into a disk with a radius of d located r units from the origin. From Eq. (2) we obtain

$$\begin{aligned} \lambda(r, d) = \frac{45}{64\pi \cdot \mathrm{E}[1/v]} \int_{\alpha_0}^{\pi} d\alpha\; d(1-x^2) \cdot \\ \int_0^{\pi} d\phi\; \sin\phi \sqrt{1 - x^2 \cos^2(\phi + \alpha - \beta)}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} x^2 &= r^2 + 2rd\cos\alpha + d^2, \\ \beta &= \arctan(r + d\cos\alpha, d\sin\alpha), \\ \alpha_0 &= \begin{cases} 0, & \text{when } r + d < 1, \\ \arccos \frac{1 - r^2 - d^2}{2rd}, & \text{when } r - d < 1 \le r + d, \\ \pi & \text{otherwise.} \end{cases} \end{aligned}$$

[1]Note that $h(\mathbf{r}, \phi)$ is symmetric with respect to ϕ, $h(\mathbf{r}, \phi) = h(\mathbf{r}, \phi + \pi)$.

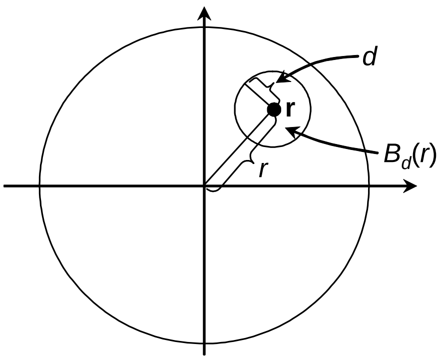

Fig. 1. Illustration of the notation.

For the special case $r = 0$ we have $x = d$, $\alpha_0 = 0$ and $\alpha = \beta$ yielding

$$\lambda(d) = \frac{45 \cdot d(1 - d^2)}{64 \cdot \mathrm{E}[1/v]} \int_0^{\pi} \sin\phi \cdot \sqrt{1 - d^2\cos^2\phi}\, d\phi. \quad (5)$$

III. Analytical Approximations for Connectivity

A. Approximations for Probability of Connectivity

We study k-connectivity and focus on the case where the movement of the nodes is restricted to a unit disk. In particular, we are interested in finding the probability that a network with n nodes is k-connected at an arbitrary point of time and denote this by $C_{n,k}(d)$, where d is the transmission range. Due to the assumed circular shape of $\mathcal{A}$, the distribution of the node location depends only on the distance $r = |\mathbf{r}|$ from the center, as given by Eq. (3). The coverage area of each node is also assumed to be circular with a radius of d and is denoted by $B_d(\mathbf{r})$, see Figure 1. Note that in principle, the domain of movement can be any convex region, and our general result (1) on the pdf $f(\cdot)$ holds. The approximations presented below depend on the shape of the domain through $f(\cdot)$ and thus hold for any convex region.

Approximation 1: We first derive the probability that an arbitrary node has at least k neighbors and denote this probability by $Q_{n,k}(d)$. Consider an arbitrarily chosen node and condition on its location, denoted by $\mathbf{r}$. Let $p(r,d)$ denote the probability that a given node is within $B_d(\mathbf{r})$ (see Figure 1), where we emphasize that this probability depends only on the distance $r = |\mathbf{r}|$ from the center. We can express $p(r,d)$ as

$$p(r,d) = \int_{\mathbf{x} \in B_d(\mathbf{r})} f(|\mathbf{x}|)\, dA,$$

where $\mathbf{x}$ denotes the vector for the location of a point inside $B_d(\mathbf{r})$. For completeness, an algorithm to compute $p(r,d)$ numerically has been given in the Appendix.

With a probability of $1 - p(r,d)$ the arbitrary node is outside $B_d(\mathbf{r})$. Since all nodes are independent, the number of other nodes within domain $B_d(\mathbf{r})$ obeys a binomial distribution, $N_{r,d} \sim \mathrm{Bin}(n-1, p(r,d))$, and hence the probability that a given node is connected to at least k nodes equals

$$1 - \sum_{i=0}^{k-1} \binom{n-1}{i} \cdot p(r,d)^i \cdot (1 - p(r,d))^{n-1-i}.$$

On the other hand, with the RWP model in the unit disk the probability density that a node is at a distance r from the center equals $2\pi r f(r)$. Thus, we can conclude that $Q_{n,k}(d)$, the probability that an arbitrary node has at least k neighbors, is given by

$$Q_{n,k}(d) = 2\pi \int_0^1 r f(r) \Bigg(1 - \sum_{i=0}^{k-1} \binom{n-1}{i} \cdot p(r,d)^i \cdot (1 - p(r,d))^{n-1-i} \Bigg) dr, \quad (6)$$

which is an exact result. As in [11], we approximate k-connectivity by

$$C_{n,k}(d) = \mathrm{P}\{n \text{ nodes are } k\text{-connected}\} \approx (Q_{n,k}(d))^n. \quad (7)$$

Note that for $n = 2$ and $k = 1$ one should use the exact result $C_{2,1}(d) = Q_{2,1}(d)$ given by Eq. (6) instead of the approximation.

The formal motivation of this approximation is as follows. As remarked in [7], for uniformly distributed random points, the asymptotics of the greatest edge length in the nearest neighbor graph are as if the nearest-neighbor distances were independent, and the longest edge is likely to be the same for the nearest neighbor graph and the minimum spanning tree. Because this holds for normally distributed points [9], the same can be expected to hold for more general spatial distributions. Here, we make the additional assumption that this generalizes to k-connectivity and the k-nearest neighbor graph.

Note that $Q_{n,k}(d)$ can, as a function of d, be interpreted as the cumulative distribution function of a single k-nearest-neighbor distance. Hence $(Q_{n,k}(d))^n$ is the cumulative distribution of the maximum of n such i.i.d. k-nearest-neighbor distances, and by the above, this is approximated to be the distribution for the greatest k-nearest-neighbor distance. The final approximation then sets this distribution equal to that of the threshold range for k-connectivity.

Approximation 2: A more simple approximation can be developed by also making an approximation in computing the probability that a certain number of nodes exist within the coverage area of a given node at location r. More specifically, we make a local Poisson assumption and assume that the nodes within the coverage area $B_d(\mathbf{r})$ result from a homogeneous Poisson point process with intensity $\lambda = f(\mathbf{r})$, i.e., the number of nodes within $B_d(\mathbf{r})$ obeys a Poisson distribution with mean equal to λ times the area of $B_d(\mathbf{r})$. Thus, within a small disk around point $\mathbf{r}$ the approximation is accurate.

Similarly as in the case of Approximation 1, we condition on the location of a single node, and thus have $n - 1$ other nodes left. Hence, we have a superposition of $n - 1$ identical Poisson point processes yielding a total intensity of $(n-1) \cdot f(\mathbf{r})$ per unit area. Consequently, the number of nodes occurring within the coverage area $B_d(\mathbf{r})$ obeys a Poisson distribution with parameter

$$a(r) = (n-1)\pi d^2 \cdot f(r), \quad (8)$$

and the probability that the number of nodes within $B_d(\mathbf{r})$ is less than k is given by $\sum_{i=0}^{k-1} \frac{a(r)^i}{i!} \cdot e^{-a(r)}$. Thus, in a unit disk our approximate probability $\hat{Q}_{n,k}(d)$ for the probability that a given node has at least k neighbors equals

$$\hat{Q}_{n,k}(d) = 1 - 2\pi \int_0^1 r\, f(r) \sum_{i=0}^{k-1} \frac{a(r)^i}{i!} \cdot e^{-a(r)}\, dr. \qquad (9)$$

In the above, it is assumed that the coverage area is a full circle even on the border of the RWP domain. The limiting effect of the border can be taken into account by introducing a function $A(r, d)$ which gives the area of the intersection of the unit disk and a disk with radius d at a distance of r from the origin, $A(r, d) = ||B_1(0) \cap B_d(r)||$. The exact form of $A(r, d)$ is given in the Appendix. With this notation the slightly more accurate approximation for $a(r)$ can be expressed as

$$a(r) = (n - 1) \cdot A(r, d) \cdot f(r). \qquad (10)$$

Finally, we use the same approximation for the probability of k-connectivity as in Eq. (7), i.e., $C_{n,k}(d) \approx (\hat{Q}_{n,k}(d))^n$.

B. Length of Connectivity Periods

Another related and important performance measure is the mean time the network remains connected. To this end, we derive analytical approximations for the mean length of the time periods the network is 1-connected. Let the random variable T_c denote the length of the time period the network is connected. Similarly, let random variable T_d denote the length of the time period the network remains unconnected. With this we have an elementary relation,

$$C_{n,1}(d) = \mathrm{P}\{n \text{ nodes are 1-connected}\} = \frac{\bar{T}_c}{\bar{T}_c + \bar{T}_d}. \qquad (11)$$

Hence, as we are interested in $\bar{T}_c$, knowledge of $C_{n,1}(d)$ and $\bar{T}_d$ is sufficient also.

If d is small the network is disconnected with a high probability and the network consists of isolated nodes or groups of connected nodes. As d increases beyond a critical value that depends on n the probability of connectivity starts increasing quickly. In practise, this is perhaps the most interesting region, and when n is large, typically only one (or few) nodes are separated from the rest of the network at the instant of time when the network becomes unconnected. Thus, we suggest estimating $\bar{T}_d$ by considering the mean interarrival time of a node into a disk $B_d(\mathbf{r})$ having a radius of d and center r units away from the center of the unit disk. Recall that, $\lambda(r, d)$ denotes the arrival rate of a single node into a disk $B_d(\mathbf{r})$ when the node moves according to RWP model in unit disk. Using either Eq. (4) or Eq. (5), as the case may be, one can compute a numerical value for $\lambda(r, d)$.

Let $\bar{T}_d^{(r)}$ denote the mean disconnectivity time on condition that a single node gets isolated at point r, which we can estimate by

$$\bar{T}_d^{(r)} \approx \hat{T}_d^{(r)} = \frac{1}{(n-1) \cdot \lambda(r, d)}.$$

Next we approximate $\bar{T}_d$ by $\hat{T}_d^{(r)}$ with some r,

$$\bar{T}_d \approx \hat{T}_d^{(r)} \qquad (12)$$

or in general case by the integral

$$\bar{T}_d \approx \int_r T_d^{(r)} \cdot g(r)\, dr, \qquad (13)$$

where $g(r)$ corresponds to the probability that the isolated node is located at the distance of r from the origin. Note that in both Eq. (12) and Eq. (13) we have made on assumption that disconnectivity is due to one isolated node. In Eq. (12) we are parameterizing the approximation with respect to the distance r from the center, and in the numerical experiments we use $r = 0$ and $r = 1$, which imply that we assume that the most likely way a network becomes disconnected is that a single node gets isolated either at the center $(r = 0)$ or on the border $(r = 1)$. In (13) we assume some distribution for the location of the isolated node, and in the numerical experiments we use the uniform distribution, $g(r) = 1/\pi$.

Finally, combining the above with Eq. (7) gives us an estimate for the mean connectivity period $\bar{T}_c$

$$\bar{T}_c = \frac{C_{n,1}(d)}{1 - C_{n,1}(d)} \bar{T}_d \approx \frac{p^n}{1 - p^n} \cdot \hat{T}_d, \qquad (14)$$

where p denotes the probability that a node has at least one neighbor, $p = Q_{n,1}(d)$ and $\hat{T}_d$ is given by either Eq. (12) or Eq. (13).

IV. Numerical Examples

A. Validation of the Probability of Connectivity

First we compare the accuracy of the approximations for 1-connectivity as a function of the radius of the coverage area d for different values of the number of nodes n. From this point on, we refer as A1 to Approximation 1 as defined in Section 3.1. Approximation 2 (in Section 3.1) actually contains two approximations and they are referred to as A2a and A2b, where A2a refers to the approximation with $a(r)$ given by Eq. (8), i.e., the domain $B_d(\mathbf{r})$ is a full circle even at the border, and A2b refers to the approximation with $a(r)$ given by Eq. (10), i.e., the border effect is taken into account. The results are shown in Figure 2, where the dashed lines correspond to simulated results and solid lines represent the analytical approximations (which approximation is in question is indicated in the figures). As can be seen A1 is remarkably accurate as n increases. Also, both A2a and A2b are able to predict well the initial rise in the value of $C_{n,k}(d)$, but they do not rise as steeply as they should as d increases. Somewhat surprisingly, the more detailed approximation A2b which includes the proper handling of the border effect, is even less accurate than the simpler A2a.

Then we validate our results for 2- and 3-connectivity. The results are shown in Figure 3, where in each figure we show simultaneously the results for 1-, 2- and 3-connectivity as a function of d for different values of n. In the simulations, the k-connectivity of the network has been determined using the algorithms described in [12]. The results only compare the accuracy of A1 (solid lines) to simulated results (dashed lines) as the accuracy of A2a and A2b is similar to that already shown before. Again, it can be seen that A1 very closely approximates the simulated values as n increases. Also, the higher the value of k the better the fit.

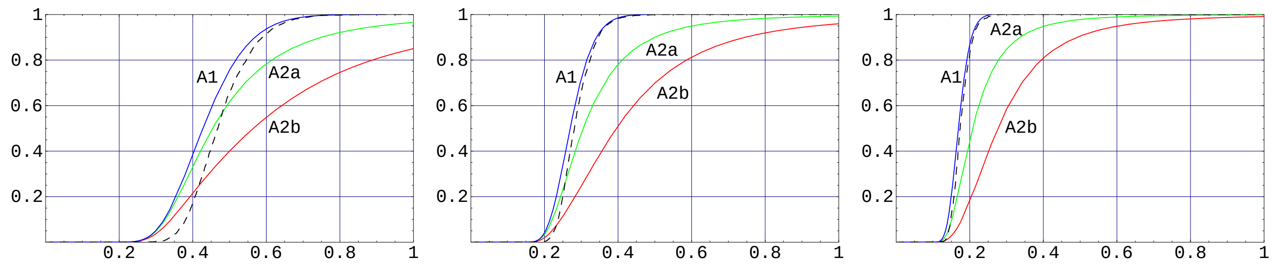

Fig. 2. Validation of 1-connectivity for $n = 20, 100, 500$ nodes (from left to right) as a function of d, dashed lines depict simulations and solid lines analytical results.

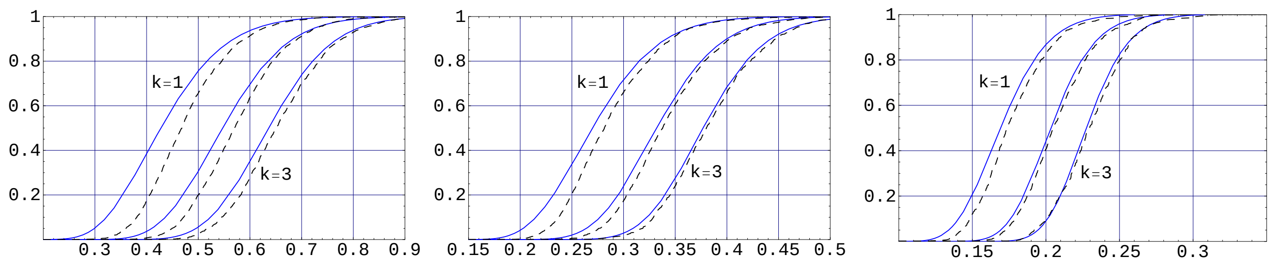

Fig. 3. Validation of k-connectivity for $n = 20, 100, 500$ nodes (from left to right) as a function of d, dashed lines depict simulations and solid lines analytical results.

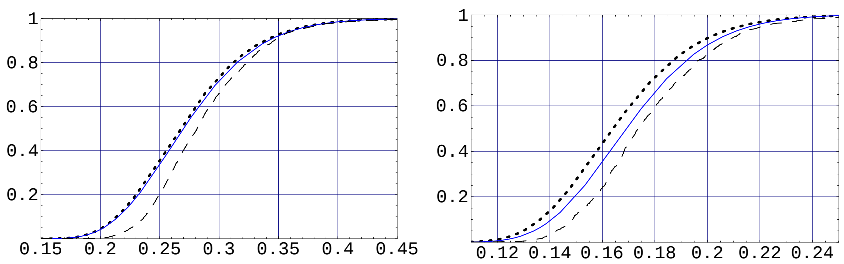

Fig. 4. Comparison of the accuracy of A1 (solid lines) and the approximation in [11] (dotted lines) against simulations (dashed lines) for $n = 100, 500$ (left, right) nodes.

Finally, we compare the accuracy of A1 with the approximation given in [11], where the aim has been to study connectivity in large networks. Figure 4 shows the results of 1-connectivity for A1 (solid lines), the approximation from [11] (dotted lines) and simulations (dashed lines) for $n = 100$ nodes (left figure) and $n = 500$ nodes (right figure). As can be seen, A1 is more accurate, especially for $n = 500$.

B. Comparison with Uniform Node Distribution

Next we compare the impact on 1-connectivity of a uniform node location distribution vs. the RWP node location distribution. The analytical results for the RWP case correspond to approximation A1, and the results for the uniform case are obtained from A1 by using $f(r) = 1/\pi$ and thus $p(r, d) = A(r, d)/\pi$. The results are shown in Figure 5, where the figure on the left contains results obtained by using our analytical approximations, and the figure on the right contains the corresponding simulated results. Each figure depicts $C_{n,1}(d)$ as a function of d for $n = 20, 100, 500$. Solid lines correspond to connectivity under RWP node distribution and dashed lines to connectivity under uniform node distribution. It can be seen that the mobility induced by the RWP model can either improve or degrade the connectivity probability depending on the number of nodes. In particular, for small number of nodes, connectivity properties gain from mobility. However, as the number of nodes is increased, the situation becomes the opposite, i.e., the required transmission range d is higher for nodes moving according to RWP than for uniformly distributed nodes (see results for $n = 500$). This phenomenon occurring in the simulations (right figure) is also captured by our analytical approximations (left figure), although numerical accuracy is not perfect for small number of nodes.

C. Mean Length of Connectivity Periods

In Figure 6 the estimated mean length of the connectivity periods are depicted as a function of d and compared against simulations, when the speed is constant, $v = 1$, and the number of nodes $n = 20, 100, 500$. Simulation results are indicated with black dashed lines and triangle markers. Blue lines with square markers correspond to our approximation where we have assumed an uniform location for isolated node. Green lines with star markers correspond to our approximation with $\bar{T}_d \approx \hat{T}_d^{(0)}$, i.e., that a node becomes most likely disconnected in the center. Red lines with diamond markers correspond to

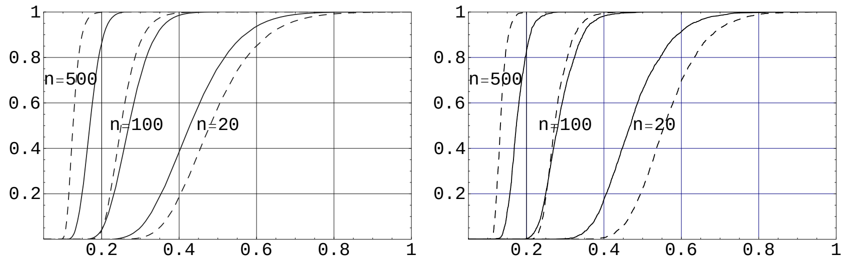

Fig. 5. Comparisons for $C_{n,1}(d)$ with RWP node distribution (solid lines) and uniform node distribution (dashed lines) using our approximations (left) and simulations (right).

our approximation with $\bar{T}_d \approx \hat{T}_d^{(1)}$, i.e., that a node becomes most likely disconnected on the border. The results show that in the interesting region, where connectivity probability rises steeply, using $\bar{T}_d \approx \hat{T}_d^{(0)}$ and $\bar{T}_d \approx \hat{T}_d^{(1)}$ gives estimates of the lower and upper bounds for the mean connectivity durations. Finally, the approximation with uniform assumption for the isolated node gives a rather accurate approximation of the mean connectivity periods for $n = 100$ and $n = 500$, where for $n = 100$ the results practically coincide. Figure 7 shows the same in a logarithmic scale, where dashed lines correspond to simulations and the solid lines represent our approximations in the same order as earlier.

D. Velocity Distributions

Next we will study how the velocity distribution affects the mean length of the connectivity period. Note that as the quantity $\lambda(r, d)$ is inversely proportional to quantity $\text{E}[1/v]$, our approximation Eq. (14) is directly proportional to quantity $\text{E}[1/v]$. In Fig. 8 the simulation results with three different velocity distributions are illustrated for $n = 20, 100, 500$ nodes, i) $v = 1$ (i.e., constant), ii) $v \sim \text{U}(0.1, 1.9)$ (i.e., $\bar{v} = 1$), and iii) $v \sim \text{U}(0.356, 2.156)$ (i.e., $\text{E}[1/v] \approx 1$). Red lines with diamond markers correspond to i), green lines with star markers correspond to ii), and blue lines with square markers correspond to iii). It can be seen that with $n = 20,\ 100,\ 500$ nodes i) and iii) are almost identical, while case ii) generally leads to longer connectivity durations. Also note that the relative difference in the results for the case ii) and cases i,iii) is close to $\text{E}[1/v] \approx 1.64$, as predicted by our approximation approach.

V. Conclusions

In this paper, we have studied the connectivity properties of ad hoc networks where the nodes are moving independently according to the RWP mobility model. Analytical approximations have been given for estimating the probability that a network consisting of n nodes is k-connected. The approximations are based on estimating the probability that the network has minimum degree k. This requires knowledge of the probability that a given node has k neighbors. The most straightforward approximation for this follows from making additional assumptions that the number of neighboring nodes obeys a Poisson distribution with an intensity depending on the location. However, by using our recent results on the exact node location distribution, we are able to compute the probability of having k neighbors exactly, which yields a very accurate approximation for the probability of k-connectivity.

The mean lengths of the 1-connectivity periods have been also studied. The approximations utilize new results on the arrival rate of the RWP process in a given subset of the movement area. These combined with our approximation for the probability of 1-connectivity yield a parameterized approximation for the mean connectivity periods. The approximation essentially represents a conditioning on the location where a single node becomes isolated from the rest of the network. The numerical results show that in the interesting region where connectivity probability rises steeply, assuming that the node gets isolated in the center or the border gives estimates of the lower and upper bounds for the mean connectivity durations. Finally, the approximation with uniform assumption for the isolated node seems to give a rather accurate approximation of the mean connectivity periods. Furthermore, according to our approximation the mean length of the connectivity period is directly proportional to quantity $\text{E}[1/v]$, which matches well with the numerical experiments.

As part of future work one can consider more accurate approximations to the mean connectivity lengths. Also, generalization of the approximations for the lengths of k-connectivity periods can be studied.

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References

[1] C. Bettstetter, G. Resta, and P. Santi, "The node distribution of the random waypoint mobility model for wireless ad hoc networks," *IEEE Transactions on Mobile Computing*, vol. 2, no. 1, pp. 25–39, 2003.

[2] J. Yoon, M. Liu, and B. Noble, "Random waypoint considered harmful," in *Proceedings of IEEE INFOCOM*, San Fransisco, California, USA, April 2003, pp. 1312–1321.

[3] W. Navidi and T. Camp, "Stationary distributions for the random waypoint mobility model," *IEEE Transactions on Mobile Computing*, vol. 3, no. 1, pp. 99–108, January-March 2004.

[4] Esa Hyytiä, Pasi Lassila, and Jorma Virtamo, "Spatial node distribution in the random waypoint mobility model with application," submitted for publication, available at `http://www.netlab.tkk.fi/julkaisut/bib/`, Mar. 2005.

[5] Esa Hyytiä and Jorma Virtamo, "Random waypoint mobility model and cellular networks," submitted for publication, available at `http://www.netlab.tkk.fi/julkaisut/bib/`, Sept. 2004.

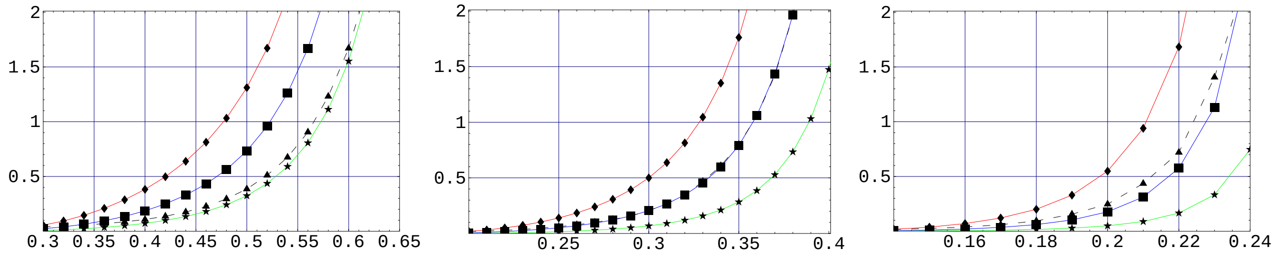

Fig. 6. Mean connectivity period length for $n = 20,\ 100,\ 500$ nodes (from left to right). Dashed curves corresponds to simulated results and solid curves to estimates.

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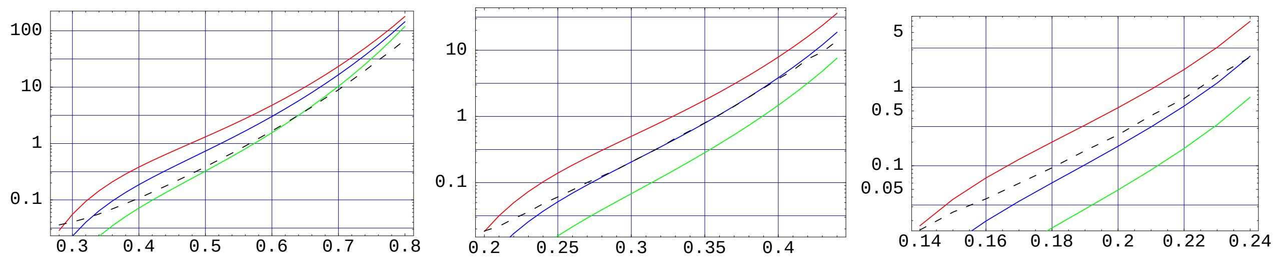

Fig. 7. Mean connectivity period length for $n = 20,\ 100,\ 500$ (from left to right) in logarithmic scale. Dashed lines correspond to simulations and solid lines to estimates.

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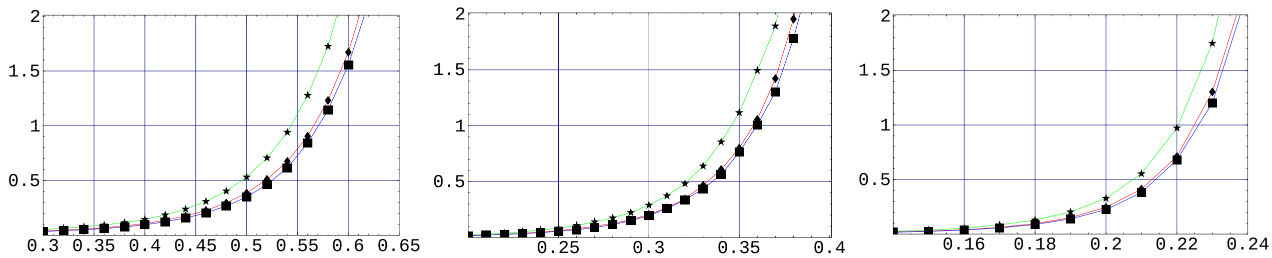

Fig. 8. Mean connectivity period length for $n = 20,\ 100,\ 500$ nodes (from left to right) with different velocity distributions.

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[6] M. Sánchez, P. Manzoni, and Z. J. Haas, "Determination of critical transmission range in Ad-Hoc networks," in *Proceedings of Multiaccess Mobility and Teletraffic for Wireless Communications 1999 Workshop (MMT'99)*, Oct. 1999.

[7] M. D. Penrose, "The longest edge of the random minimal spanning tree," *Annals of Applied Probability*, vol. 7, no. 2, pp. 340–361, 1997.

[8] M. D. Penrose, "On k-connectivity for a geometric random graph," *Random Structures and Algorithms*, vol. 15, no. 2, pp. 145–164, 1999.

[9] M. D. Penrose, "Extremes for the minimal spanning tree on normally distributed points," *Advances in Applied Probability*, vol. 30, no. 3, pp. 628–639, 1998.

[10] Peng-Jun Wan and Chih-Wei Yi, "Asymptotic critical transmission radius and critical neighbor number for k-connectivity in wireless ad hoc networks," in *Proceedings of ACM MobiHoc '04*. 2004, pp. 1–8, ACM Press.

[11] Christian Bettstetter, "On the connectivity of Ad Hoc networks," *Computer Journal*, vol. 47, no. 4, pp. 432–447, July 2004.

[12] H. Koskinen, "A simulation-based method for predicting connectivity in wireless multihop networks," *Telecommunication Systems*, vol. 26, no. 2-4, pp. 321–338, June 2004.

Appendix

A. Probability of Finding a Node inside $B_d(\mathbf{r})$

An algorithm for computing $p(r, d)$, the probability of finding a node inside a disk with a radius d at the distance of r from the origin, is given in Algorithm 1 and 2 (see also Figure 9). Recall that $h(t)$ and C are according to (1).

Algorithm 1 Function $s(r, d, t)$

if $t \le 0$ or $t \le d - r$ **then**
  $\theta = 2\pi$
else
  $A = d/t$
  $B = r/t$
  $\theta = 2(\pi/2 - \arcsin\left((1 + B^2 - A^2)/(2B)\right))$
end if
return $(1/C) \cdot \theta \cdot t \cdot h(t)$

Algorithm 2 Function $p(r, d)$

$t_0 = \max\{0,\ r - d\}$
$t_1 = \min\{1,\ r + d\}$
if $d > r$ **then**
  $x = \int_{t_0}^{d-r} s(r, d, t)\ dt + \int_{d-r}^{t_1} s(r, d, t)\ dt$
else
  $x = \int_{t_0}^{t_1} s(r, d, t)\ dt$
end if
return x

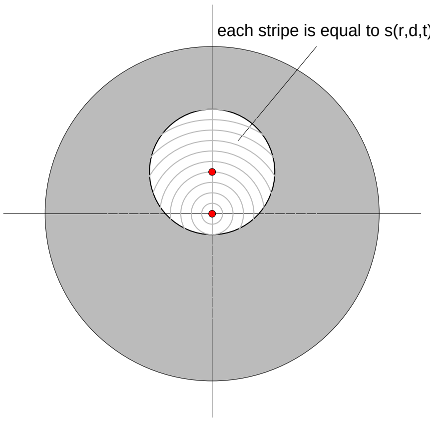

Fig. 9. Partitioning the $B_d(\mathbf{r})$ into circular "stripes" results in a one directional integral.

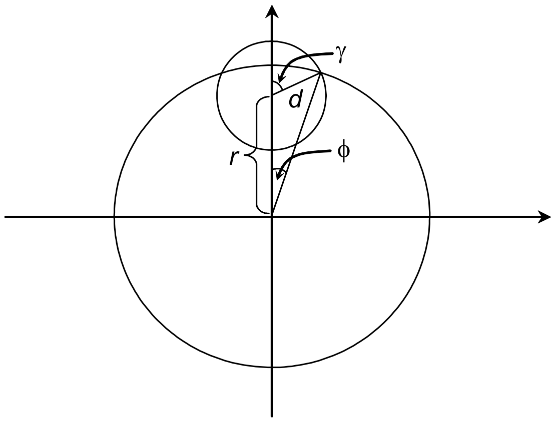

Fig. 10. Notation for used in $A(r, d)$ for unit disk.

B. Area of the Intersection of a Unit Disk and $B_d(\mathbf{r})$

From Figure 10 it is easy to see that the area of the intersection of a unit disk and $B_d(\mathbf{r})$, $A(r, d)$, equals πd^2 as long as $r \leq 1 - d$. If $r > 1 - d$, then the part of the small disk outside the unit disk is given by the difference of two segments. Hence, for the unit disk $A(r, d)$ is given by

$$A(r,d) = \begin{cases} \pi d^2, & \text{if } r \leq 1-d, \\ \pi d^2 - \big((\gamma d^2 - \frac{1}{2} d^2 \sin 2\gamma) - \\ \quad (\phi - \frac{1}{2}\sin 2\phi)\big), & \text{if } 1-d < r \leq 1, \end{cases}$$

where

$$\begin{cases} \gamma = \arccos \dfrac{1 - d^2 - r^2}{2rd}, \\ \phi = \arcsin \dfrac{\sqrt{4r^2 d^2 - (1 - d^2 - r^2)^2}}{2r}. \end{cases}$$