HELSINKI UNIVERSITY OF TECHNOLOGY | **Exercise 6**
Networking Laboratory | 11.12.2007
S-38.3143 Queueing Theory, II/2007 | Virtamo / Penttinen

1. Carloads of customers arrive at a single-server station in accordance with a Poisson process with rate 4 per hour. The service times are exponentially distributed with mean 3 min. If each carload contains either 1, 2, or 3 customers with respective probabilities $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, compute the average customer waiting time in the queue. Hint: The waiting time of the first customer of each group can be obtained from an appropriate $M/G/1$ queue. Consider separately the "internal" waiting time in the group.

2. Persons arrive at a copying machine according to a Poisson process with rate 1/min. The number of copies to be made by each person is uniformly distributed between 1 and 10. Each copy takes 3 s. Find the average waiting time in the queue when

   a) Each person uses the machine on a first-come first-served basis.
   b) Persons with no more than 2 copies to make are given non-preemptive priority over other persons.

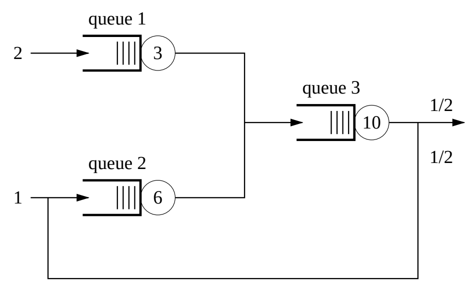
3. Consider a priority queue with two classes and preemptive resume priority. Customers arrive according to two independent Poisson processes with intensities λ_1 and λ_2. Service times in both classes are independent and exponentially distributed with a joint mean $1/\mu$. Determine the mean sojourn times $\bar{T}_1$ and $\bar{T}_2$ for both classes.

4. The Pollaczek-Khinchin formula for the Laplace transform of the waiting time W is

$$W^*(s) = \frac{s(1-\rho)}{s-\lambda+\lambda S^*(s)}$$

   where $S^*(s)$ is the Laplace transform of the service time S and $\rho = \lambda\overline{S}$. Using this result, rederive the PK mean formula for the waiting time.

5. Queues 1 and 2 of the open Jackson queueing network depicted in the figure receive Poissonian arrival streams with rates 2 and 1 (customers/s). Service times are exponentially distributed with the given rates (customers/s). Calculate a) customer streams through each of the queues, b) average occupancies of the queues and the average total number of customers in the network, c) mean delays in the network of customers arriving at queues 1 and 2 as well as the delay of an arriving customer chosen at random.



6. Consider a cyclic closed network consisting of two queues. The service times in the queues are exponentially distributed with parameters μ_1 and μ_2. There are three customers circulating in the network. a) Draw the state transition diagram of the network (four states). b) Determine the equilibrium probabilities and calculate the mean queue lengths. c) Calculate the customer stream in the network (e.g. the customer stream departing from queue 1). d) Rederive the results of b) and c) by means of the mean value analysis (MVA).