HELSINKI UNIVERSITY OF TECHNOLOGY **Exercise 5**
Networking Laboratory 4.12.2007
S-38.3143 Queueing Theory, II/2007 Virtamo / Penttinen

1. Assume that there is always one doctor at a clinic (24 hours a day) and that customers arrive according to a Poisson process with intensity λ. The service time of each customer obeys an exponential distribution with mean 30 minutes. Determine the maximum arrival intensity that allows 95% of the customers to be served within 72 hours from (the first) arrival.

2. Show that in an $M/G/1$-LIFO queue the equilibrium distribution of the queue length, $\pi_n = (1-\rho)\rho^n$, is insensitive to the holding time distribution. Hint: Reason that a customer who upon arrival finds $j-1$ customers in the system will be exclusively served when the system is in state $N = j$. Thus, the time the system spends in the state $N = j$ is completely composed of the full service times of the customers who arrive to the system in the state $N = j-1$. How many such arrivals occur in a long interval of time T?

3. Consider an M/M/1/K queue with states $0, 1, \ldots, K$. Find the probability P_n that the queue which initially is in the state n becomes empty before flowing over. Hint: Add an imaginary state $K+1$ to the system; a transition to the state $K+1$ corresponds to the queue flowing over. We have $P_0 = 1$ and $P_{K+1} = 0$. For states $n = 1, \ldots, K$, write the probability P_n in terms of P_{n-1} ja P_{n+1}. Solve the equations.

4. Customers arrive at a two-server system according to a Poisson process having rate $\lambda = 5$/min. An arrival finding server 1 free will begin service with that server. An arrival finding server 1 busy and server 2 free will enter service with server 2. An arrival finding both servers busy goes away. Once a customer is served by either server, he departs the system. The service times of the servers are exponential with rates $\mu_1 = 4$/min and $\mu_2 = 2$/min. a) What is the average time an entering customer spends in the system? b) What proportion of time is server 2 busy?

5. Customers arrive at an $M/\mathrm{Erlang}(k,\mu)/1$ system according to a Poisson process with rate λ. Find the mean waiting and sojourn times of a customer in the system?

6. If in a single server system each customer has to pay a fee to the system according to some rule, then the average revenue rate of the system = $\lambda \cdot$ (average fee), where λ is the mean rate of arriving customers.

   Apply this to the $M/G/1$ system with the following charging rule: each customer in the system pays at rate which is the same as the customer's remaining service time. What is the average fee? Show by equating the above average revenue rate with the average charging rate (time charging) that
   $$\overline{W} = \lambda\,(\overline{X}\,\overline{W} + \overline{X^2}/2),$$
   where W and X are the waiting and service times. Solve $\overline{W}$. What is this result?