HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory, II/2007

Exercise 1
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1. There are two coins one of which is normal and the other one has head on both sides. One of the coins is chosen randomly and tossed m times and each time the result is head. What is the probability that the chosen coin is the normal one. Calculate also the numerical value for $m = 1, 2, 3$.

2. In a Bernoulli trial 1 is obtained with probability p and 0 with probability $q = 1 - p$. The value of p is unknown, but it is known that it is drawn from a uniform distribution in (0,1), i.e. all values in this interval are *a priori* equally likely. The Bernoulli trial is repeated in order to estimate the real value of p. In the trials result 0 occurs n_0 times and result 1 n_1 times. What is the *posterior* distribution (Bayes) of p according to the trials? Where is the maximum of the distribution?

3. Apply the conditioning rules

$$\begin{aligned} \mathrm{E}[X] &= \mathrm{E}[\mathrm{E}[X|Y]] \\ \mathrm{V}[X] &= \mathrm{E}[\mathrm{V}[X|Y]] + \mathrm{V}[\mathrm{E}[X|Y]] \end{aligned}$$

   to the case $X = X_1 + \ldots + X_N$, where the X_i are independent and identically distributed (i.i.d.) random variables with mean m and variance σ^2, and N is a positive integer valued random variable with mean n and variance ν^2. Hint: Condition on the value of N.

4. Five different applications are transmitting packets in a LAN. For each application i, $i = 1, \ldots, 5$, measurements are carried out to determine the mean, m_i, and the standard deviation, σ_i, of the packet lengths. The measurements provide also a classification of the packets giving the proportion of packets belonging to each application i (denoted by p_i). The results of the measurements are summarized in the following table. Compute the mean and the standard deviation of the lengths of all packets in the network.

application	p_i	m_i	σ_i
1	0.30	100	10
2	0.15	120	12
3	0.40	200	20
4	0.10	75	5
5	0.05	300	25

5. Assume that the traffic process in a LAN is stationary, whence periods of equal length are statistically identical. Denote the amount of traffic (kB) carried in the network in consecutive 10 min periods by X_1, X_2 and X_3 (random variables). In measurements over a long time one has observed that the variance of the traffic in periods of 10 min, 20 min and 30 min are v_1, v_2 ja v_3, respectively. In other words, $v_1 = \mathrm{V}[X_1] = \mathrm{V}[X_2] = \mathrm{V}[X_3]$, $v_2 = \mathrm{V}[X_1 + X_2] = \mathrm{V}[X_2 + X_3]$ and $v_3 = \mathrm{V}[X_1 + X_2 + X_3]$. Determine $\mathrm{Cov}[X_1, X_2] = \mathrm{Cov}[X_2, X_3]$ and $\mathrm{Cov}[X_1, X_3]$ in terms of v_1, v_2 and v_3. Hint: e.g. the previous one can be obtained by expanding $v_2 = \mathrm{V}[X_1 + X_2] = \mathrm{Cov}[X_1 + X_2, X_1 + X_2]$.

6. The length of the packet (in bytes) arriving to a router is assumed to obey geometric distribution. The mean length of packet is 100 bytes. Each packet is first read into an input buffer. How large the input buffer should be in order that an arriving packet fits there with probability of 95 % or higher?