HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory, Fall 2005

Exercise 1
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1. There are two coins one of which is normal and the other one has head on both sides. One of the coins is chosen randomly and tossed m times and each time the result is head. What is the probability that the chosen coin is the normal one. Calculate also the numerical value for $m = 1, 2, 3$.

2. Apply the conditioning rules

$$\begin{aligned} \mathrm{E}\,[X] &= \mathrm{E}\,[\mathrm{E}\,[X|Y]] \\ \mathrm{V}\,[X] &= \mathrm{E}\,[\mathrm{V}\,[X|Y]] + \mathrm{V}\,[\mathrm{E}\,[X|Y]] \end{aligned}$$

   to the case $X = X_1 + \ldots + X_N$, where the X_i are independent and identically distributed (i.i.d.) random variables with mean m and variance σ^2, and N is a positive integer valued random variable with mean n and variance ν^2. Hint: Condition on the value of N.

3. Let $X_1,\ X_2, \ldots, X_n$ be geometrically distributed random variables, $X_i \sim \mathrm{Geom(p_i)}$. What distribution does $\min(X_1,\ X_2, \ldots, X_n)$ obey?

4. Let $S = X_1 + \ldots + X_N$, where $X_i \sim \mathrm{Exp}(\mu)$, be i.i.d. and N an independent geometrically distributed random variable, $\mathrm{P}\{N = k\} = (1-p)p^{k-1}$, $k = 1, 2, \ldots$. Determine the tail distribution of S, $G(x) = \mathrm{P}\{S > x\}$.

5. Assume that the length of a web surfing session obeys exponential distribution with mean 36 min.

   a) What is the probability that a session lasts 30 min or more?

   b) What is the probability that a session lasts at least one hour?

   c) A session has already lasted one hour. What is the probability that it lasts at least one more hour?

   d) 90 % of the sessions last less than R minutes. What is the value of R?

6. Let X be an exponential random variable. Without any computations, tell which one of the following is correct. Explain your answer.

   a) $\mathrm{E}\,[X^2|X > 1] = \mathrm{E}\,[(X+1)^2]$
   b) $\mathrm{E}\,[X^2|X > 1] = \mathrm{E}\,[X^2] + 1$
   c) $\mathrm{E}\,[X^2|X > 1] = (1 + \mathrm{E}\,[X])^2$