HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.3141 Teletraffic Theory, IV/2008

Exercise 6
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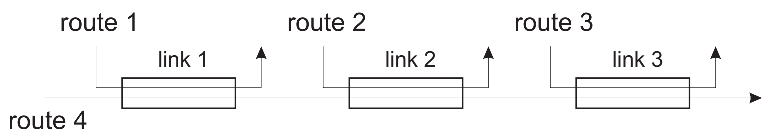
Virtamo / Penttinen

1. Show that in a slotted Aloha system with optimal retransmission probability $p_r(n) = \frac{1-\lambda}{n-\lambda}$, the drift becomes $d(n) = \lambda - e^{-\lambda}\left(\frac{n-1}{n-\lambda}\right)^{n-1}$, and furthermore that $d(n) \to \lambda - 1/e$ when $n \to \infty$.

2. Consider a base station to which n users are sending packets in synchronized timeslots. Probability that a given user transmits in a given timeslot is p (the system is often referred to as slotted Aloha). If more than one user transmits in a given time slot the base station cannot receive any of the transmissions. For which value of p the user experienced throughput (i.e., probability of successful transmission) is the largest? In that case, what is the user experienced throughput as $n \to \infty$ compared to a fully coordinated system where the users transmit in turns?

3. Consider a slotted Aloha system where two users are transmitting on a same channel. Denote the transmission probabilities of the users by p_1 and p_2. The capacity of the channel as seen by a user is the probability of successful transmission (times the nominal capacity of the channel). Determine the capacity set of the system, i.e. the set of user capacities (c_1, c_2) that can be achieved by adjusting the transmission probabilities. Hint: The user capacity lies on the interval $[0, 1]$. Determine the border of the capacity set by fixing the capacity of one user and then finding the maximum capacity of the other user.

4. A network consists of three links, $j = 1, 2, 3$, each having the capacity $C_j = 1$. The flows in class $k = 1, 2, 3$ go only through the corresponding link $j = k$ and, in addition, the flows in class $k = 4$ go through all the three links. There are k flows in each class k. Find the maxmin fair bandwidth sharing for the system.



5. In a 2-branch tree network the trunk has the capacity $C_0 = 3$ and the branches have the capacities $C_1 = C_2 = 2$ (MBit/s). Route 1 goes through the trunk and branch 1 and route 2 goes through the trunk and branch 2. Find fair bandwidth shares when there are $n_1 = 1$ connections on route 1 and $n_2 = 3$ connections on route 2 and the fairness criterion is a) maxmin fairness, b) proportional fairness.

6. In a cellular system with cell radius of 4 km, data traffic is carried using adaptive link techniques so that the feasible (unshared) link rate depends on distance as follows: 20 Mbit/s, 10 Mbit/s, 5 Mbit/s and 2 Mbit/s up to distances of 1 km, 2 km, 3 km and 4 km. Assume that the users share equally the limiting resource (transmission time of the base station) and that the destinations of the flows arriving to the base station are uniformly distributed over the area of the cell. Find the cell capacity C (Mbit/s), i.e. the maximal sustainable total bit rate of the whole cell when a) the users are stationary b) the users move randomly within the cell so fast that from the point of the transmission the link rate is constant (average over the cell).

   At the capacity limit the load of the PS queue is 1. Find also the distribution of this load between the zones in the case a).