S-38.211 Signal Processing in Communications I	Amoakoh Gyasi-Agyei
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Theme: Adaptive Equalization, MSE gradient algorithm	Email: amoakoh@luuri.hut.fi

*Note: Exercises marked * may be used for private studies.*

Figure 1 below shows an N tap FIR precursor (non-causal) equalizer

$$C(z) = \sum_{m=-L}^{L} c_m z^{-m} \qquad (L = (N\text{-}1)/2,\ N \text{ odd}).$$

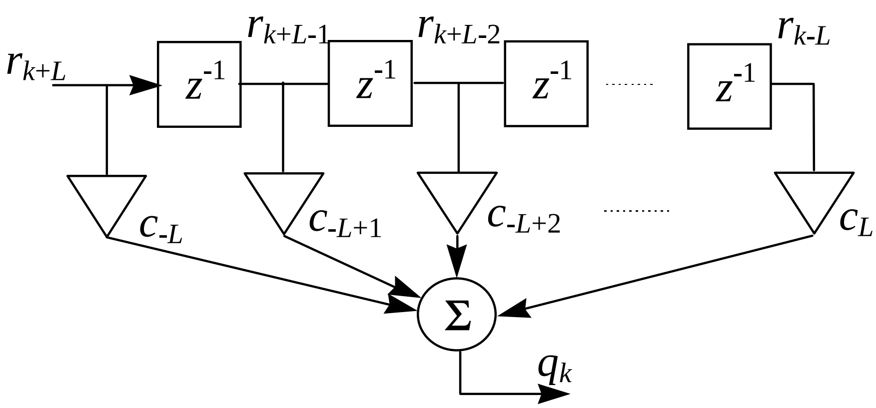

Figure 1: *N-tap FIR equalizer.*

Let us collect the filter coefficients and the input samples into the two vectors $\mathbf{c} = [c_{-L} \ \cdots \ c_0 \ \cdots \ c_L]^{\mathrm{T}}$ and $\mathbf{r}_k = [r_{k+L} \ \cdots \ r_k \ \cdots \ r_{k-L}]^{\mathrm{T}}$. We can now express the error signal at the equalizer output as $e_k = a_k - q_k = a_k - \mathbf{c}^{\mathrm{T}}\mathbf{r}_k$, where a_k is the transmitted data, and q_k is the equalizer output.

The mean-squared error at the equalizer output is defined as $MSE = E\left[|e_k|^2\right] = E\left[|a_k - \mathbf{c}^{\mathrm{T}}\mathbf{r}_k|^2\right]$. The coefficient vector **c** that minimizes the *MSE* above is referred to as the optimum vector in the *mean-squared sense*. This solution is studied in *LM-11.1.2*. Several iterative algorithms exists that tries to solve this equation. One of them is the MSE gradient (**MSEG**) algorithm (LM Ch. 11.1.3) and it is the one we are going to take a look at in this exercise/homework.
(I think that before we start, we can recapitulate some of the notations and formulas used at lectures and that's the reason for the above story !!)

Exercise 4-1

Suppose the autocorrelation matrix Φ of the sampled input data signal to an equalizer and the vector $\boldsymbol{\alpha}$ for a given experimental environment are known to be

$$\Phi = E\left[\mathbf{r}_k^* \mathbf{r}_k^{\mathrm{T}}\right] = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \quad \boldsymbol{\alpha} = E\left[a_k \mathbf{r}_k^*\right] = \begin{bmatrix} 0.5 & 0.25 \end{bmatrix}^{\mathrm{T}}$$

a) Calculate the optimum coefficient vector (Wiener solution).

b) Calculate the minimum MSE assuming that the variance of the transmitted signal $E\left[|a_k|^2\right]$ signal is given by σ_a^2.

c) For the environment described above state the update formula for the MSEG algorithm.

d) What is the maximum step-size that can be used?

***Exercise 4-2** (cf. P 11-10 of Lee & Messerschmitt)

A real-valued input WSS process X_k has the PSD (power spectral density)

$$\Phi_X(z) = \frac{A}{(1-\alpha z)(1-\alpha z^{-1})}, \quad 0 < \alpha < 1$$

and the region of convergence includes the unit circle.

(a) Find the autocorrelation function of X_k, *i.e.*, $\phi_X(k)$.

(b) Find the predictor coefficients for a minimum MSE N-th order predictor.

***Exercise 4-3** (cf. P 11-5 of Lee & Messerschmitt)

Consider an input WSS random process with ACF $\phi_Y(k) = \beta^{|k|}$.

(a) Find the power spectrum of this random process.

(b) Find the asymptotic minimum and maximum eigenvalues and eigenvectors of the autocorrelation matrix.

(c) Find, as a function of β, the eigenvalues and eigenvectors of the 2x2 autocorrelation matrix.

(d) Find the eigenvalue spread of the autocorrelation matrix by the approximate relation

$$\lambda_{\max} = \max_{\omega} \Phi\left(e^{j\omega}\right), \qquad \lambda_{\min} = \min_{\omega} \Phi\left(e^{j\omega}\right)$$

and compare to the results of (c).

(e) Find, as a function of β and as $N \to \infty$, the step size μ and the resulting dominant mode of convergence of the MSEG algorithm. Interpret this results.

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Homework 4 (Submission Deadline: November 4, 1998 at 11.15 am) - **10 points**

Suppose the autocorrelation matrix ,Φ, of the sampled input data signal to an equalizer, and the vector α for a given experimental environment are known to be

.

$$\Phi = E\left[\mathbf{r}_k \mathbf{r}_k^{\mathrm{H}}\right] = \frac{1}{4}\begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 4 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \qquad \alpha = \begin{bmatrix} \frac{1}{2} & \frac{3}{8} & \frac{2}{8} & \frac{1}{8} \end{bmatrix}^{\mathrm{T}}$$

a) Calculate the optimum filter coefficient vector (Wiener solution). (2p)

b) Calculate the minimum MSE considering that the variance of the reference signal is equal to one (i.e., $\sigma_a^2 = 1 = E\left[\left|a_k\right|^2\right]$). (2p)

c) For the environment described above state the update formula for the MSEG algorithm. (2p)

d) Assuming that the adaptive filter coefficients are initially zero, calculate their value for the first ten iterations. (2p)

e) Find the eigenvalues and eigenvectors of the autocorrelation matrix. (2p)