S-38.211 Signal Processing in Communications I	Amoakoh Gyasi-Agyei
Tutorial 3 7/10/1998	Phone: 451 2204 Room: I208
Theme: Equalizers, MSE	Email: amoakoh@tct.hut.fi

Problems marked * are meant for private studies

Figure 1 below illustrates a discrete-time model of a digital communication system where the channel noise n_k is modeled as an additive white Gaussian noise (AWGN).

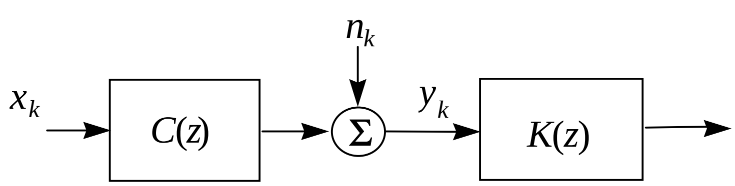


Figure 1: *Block diagram of the communication system.*

Exercise 3-1

Since the channel is not ideal it causes ISI. Thus the channel can be modeled with the discrete-time transfer function $C(z) = 1 + \alpha z^{-1}$. The random input sequence is iid and assumes the values $x_k \in \pm 1$.

a) Assume that α is a positive constant smaller than unity (i.e., $0 < \alpha < 1$). Draw the pole-zero diagram of $C(z)$. Find the optimal linear zero-forcing (LE-ZF) equalizer $K(z)$ for this channel and draw the filter structure.

b) Assume that $|\alpha| > 1$. Draw the pole-zero diagram of $C(z)$. Find the optimal LE-ZF equalizer $K(z)$ (which is minimum-phase to guarantee stability) and draw the filter structure.

c) Show that the magnitude of the frequency responses of the two equalizers are the same (regardless of the value of α).

d) Compute the equalizer output noise power spectrum for case (a) (assuming that the noise at the equalizer input is white with the variance σ_n^2) and determine the noise power (variance).
Hint: Use Parseval's relation and / or the superformula.

***Exercise 3-2**

Given an AWN transmission channel with the transfer function $H(z) = 1/(1-cz^{-1})$:

(a) When $|c| < 1$, find the precursor and postcursor equalizers and the resulting MSE.

(b) Repeat (a) for $|c| > 1$.

(c) Interpret the results of (a) and (b).

S-38.211 Signal Processing in Communications I	Amoakoh Gyasi-Agyei
Tutorial 3 7/10/1998	Phone: 451 2204 Room: I208
Theme: Equalizers, MSE	Email: amoakoh@tct.hut.fi

Exercise 3-3

Determine the optimum coefficients of the linear equalizer (predictor) depicted in Fig. Ex 3.3. Here $x(n)$ is the input to the equalizer/predictor filter, and $e(n)$ is the prediction error signal that should be minimized to obtain the optimum predictor coefficients.

(a) Find the optimum predictor coefficients a_k.

(b) Sketch the simplified predictor structure which results from (a).

(c) What is the optimal MSE?

S-38.211 Signal Processing in Communications I	Amoakoh Gyasi-Agyei
Tutorial 3 7/10/1998	Phone: 451 2204 Room: I208
Theme: Equalizers, MSE	Email: amoakoh@tct.hut.fi

Homework 3 (Return time: October 28, 1998)

A communications channel that introduces ISI has the z transform $C(z) = \frac{1}{1+\beta z^{-1}}$

a) Assume that β is a positive constant smaller than unity (i.e., $0 < \beta < 1$). Draw the pole-zero diagram. Find the optimal linear zero-forcing (LE-ZF) equalizer $K(z)$ and draw the filter structure.

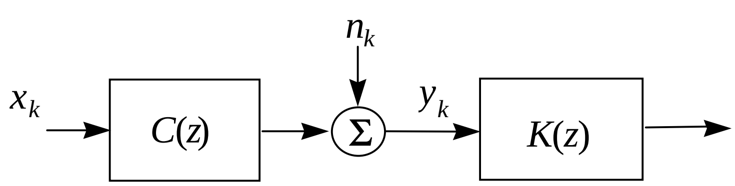
b) Now assume that $\beta > 1$. Draw the pole-zero diagram. Find the optimal LE-ZF equalizer $K(z)$ (which is minimum-phase) and draw the filter structure. Is it a problem here to have a maximum-phase equalizer? Why?

c) Show that the magnitude of the frequency responses of the two equalizers are the same (regardless of the value of β).

d) Compute the output noise power spectrum (assuming that the noise of the equalizer input is white) and determine the noise power (variance).

e) Find the resulting MSEs (mean square errors) in (a) and (b), i.e., compute ε^2_{LE-ZF} .

NOTE: The random input sequence is iid and assumes the values $x_k \in \pm 1$.



Homework: *Block diagram of the communication system.*