## Exercise 2-1

Consider transmitting bits  $X_k$  (zeros and ones) over a channel with additive white Gaussian noise. Assume that  $X_k = 0$  for  $k \le 0$  and  $k \ge K$ . Suppose that K = 3 and the observation sequence is  $\{y_0, y_1, y_2, y_3\} = \{0.6, 0.9, 1.3, 0.3\}$ . The model for the ISI channel is  $g_k = \delta_k + 0.5\delta_{k-1}$ 

- a) Model the system as a shift register process and draw the state transition diagram. Label the arcs with the input/output pairs ( $X_k$ ,  $S_k$ ).
- b) Draw one stage of the trellis and label with the input/output pairs  $(X_k, S_k)$ .
- c) Draw the trellis for the Markov model and label the transition weights. What is the ML detection of the incoming bit sequence?
- d) Find the ML decision sequence  $\hat{x}_k$  assuming that the additive noise is the only degradation (no ISI) and that  $X_k$  are iid (independent, identically distributed) random sequences.

## Exercise 2-2

A convolutional code is given by:

$$S_{\kappa}^{1} = X_{\kappa} + X_{\kappa-2}$$
$$S_{\kappa}^{2} = X_{\kappa} + X_{\kappa-1} + X_{\kappa-1}$$

where the output of the encoder at a discrete time instant k is given by:  $C_{K} = S_{k}^{1}S_{k}^{2}$ 

- (a) Draw the block diagram of the encoder
- (b) Model the system with shift registers and draw the state transition diagram with complete labeling.
- (c) Draw the trellis diagram (code tree) for the length L = 2. (Append as many zeros as needed to the data bits.)
- (d) Decode the following sequence using the **Viterbi algorithm** (or ML decoder which is optimal for an AWGN channel): 11 10 11 00.

## Homework 2 (Submission Deadline: Wednesday, October 21, 1998 at 11.15 am)

Assume  $X_k$  is equally likely to be 0 or 1, and the  $X_k$  are independent for all k. Assume additional additive Gaussian white noise with variance  $\sigma^2$ . The ISI channel is given as  $g_k = \delta_k - 0.5\delta_{k-1} + 0.1\delta_{k-2}$ .

- a) Model the system as a shift register process and draw the state transition diagram. Label the arcs with the input/output pairs ( $X_k$ ,  $S_k$ ).
- b) Draw one stage of the trellis and label with the input/output pairs  $(X_k, S_k)$ .
- c) Assume  $\Psi_k = 0$  for  $k \le 0$  and  $k \ge 5$ . Suppose the observation sequence is  $\{y_0, y_1, y_2, y_3, y_4\} = \{0.5, -0.2, 0.9, 1.2, 0.1\}$ . Draw a complete trellis with branch weights labeled.

- d) Use the Viterbi algorithm to find the ML decision sequence!
- e) What is the ML decision sequence  $\hat{x}_k$  assuming only additive noise (no ISI)?