S-38.211 Signal Processing in Communications I	Amoakoh Gyasi-Agyei
Tutorial 1, 30/9/1998	Phone: 451 2204 Room: I208
Theme: Matched Filtering, SNR, Nyquist Criterion, ISI, Eye pattern	Email: amoakoh@luuri.hut.fi

NOTE: ***The additional exercises marked with asterisk "*" may be used for private studies.***

Figure 1 below shows a simple model of a binary PAM-system:

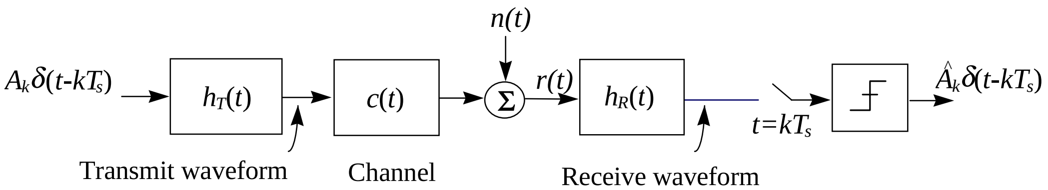

Figure 1: *Model of a binary PAM system*

The data $A_k \in \{\pm 1\}$ consists of independent identically distributed binary symbols, $h_T(t)$ is the transmit filter, $h_R(t)$ is the receiver filter, $c(t)$ is the channel impulse response, and $n(t)$ is AWGN with the double sided power spectral density $N_0/2$. Assuming an ideal channel, $c(t)$, the received signal $r(t)$ can be expressed as

$$r(t) = \sum_{m=-\infty}^{\infty} A_m h_T(t - mT_s) + n(t) .$$

In these exercises we will study different choices of the filters $h_T(t)$ and $h_R(t)$.

Exercise 1-1 (IFT, Nyquist Criterion, ISI)

The spectra $H_{T1}(f)$ and $H_{T2}(f)$ of two pulse shaping filters $h_{T1}(t)$ and $h_{T2}(t)$, respectively, are shown in Figure 2.

a) Find the corresponding transmit pulse shapes h_{T1}(t) and h_{T2}(t).
b) Show that the two pulses satisfy the Nyquist criterion, both in time and frequency domains.
c) Can you sketch some other spectra that satisfy the Nyquist criterion?
d) What is the importance of the Nyquist criterion?

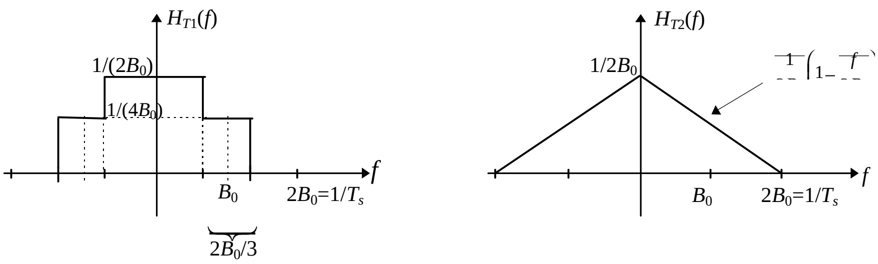

Figure 2: *Spectra of the pulse shapes in Exercise 1-1*

***Exercise 1-2** (Matched Filtering, SNR)

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The unnormalized matched filter for the signal $x(t) = A \cdot rect\left(t/T - 1/2 \right)$ has the impulse response $g(t) = \frac{1}{T} \cdot rect\left(1/2 - t/T \right)$. WGN (white Gaussian noise) $w(t)$ with the double sided PSD (power spectral density) $N_0 / 2$ also enters at the filter input. The signal-to-noise ratio (SNR) at the filter output is defined as

$$SNR = \frac{|y(T)|^2}{\sigma^2} \quad \text{(i.e., the peak pulse SNR)}$$

where $y\,(T)$ is the output signal at the time instant T with $x(t)$ as input signal, and σ^2 is the noise power at the filter output.

(a) Sketch the output signal $y(t)$ (neglect the noise term), where $x(t)$ is the input signal.

(b) Compute the SNR for the filter above.

(c) Assuming that the matched filter above is replaced by the following suboptimal filter

$$g(t) = \begin{cases} \frac{1}{T} e^{-\alpha t} & t > 0, \alpha = \frac{1.25}{T} \\ 0 & \text{otherwise} \end{cases}$$

compute the SNR for this situation. What conclusions can be drawn from (b) and (c)?

(d) Why do we use matched filters?

Exercise 1- 3 (Matched filtering)

Consider the signal $x(t)$ shown in Fig. E1-3.

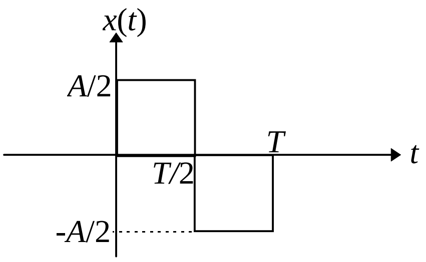

Fig. E1-3

(a) Determine the impulse response of a filter $h(t)$ matched to this signal and sketch it as a function of time.

(b) Plot the matched filter output $y(t)$ as a function of time.

(c) What is the peak value of the output? How is it related to the input pulse energy?

***Exercise 1- 4** (Eye pattern / diagram)

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The two figures below depict two different eye patterns of received signals at a digital communications receiver that monitors the effects of noise and ISI on optimal reception.

(a) Give the number of signaling levels or the number of bits transmitted per symbol duration in both systems.

(b) Indicate in each diagram the optimal point to sample the received signal.

(c) Justify your decisions in (b).

(d) What is eye pattern diagram and how can it be produced on oscilloscope?

(e) What effect has bandwidth on the appearance of the eye pattern diagram?

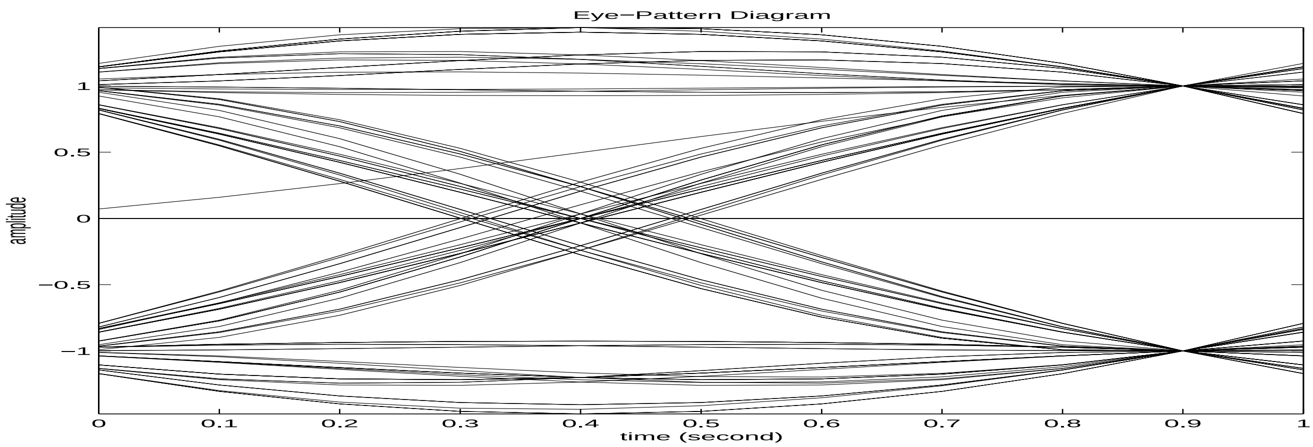

Figure 1 Eye pattern for exercise E 1-4

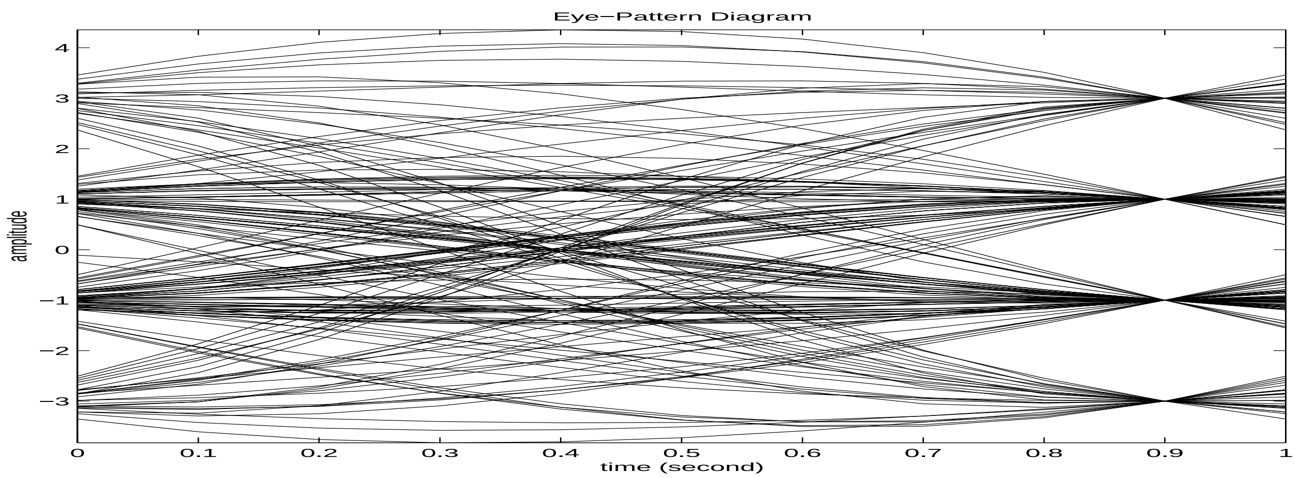

Figure 2 Eye pattern for exercise E 1-4

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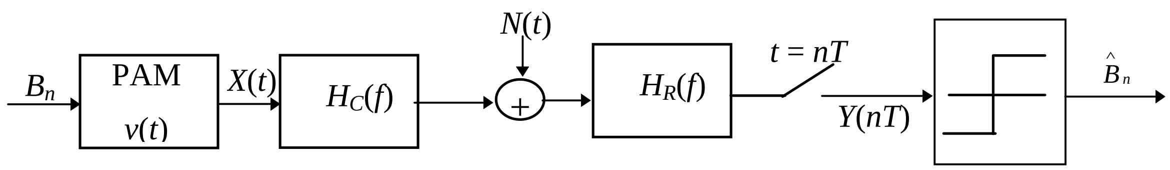
HOMEWORK 1 (10 points)

Submission Deadline: Wednesday, October 7, 1998 at 11.15am.:

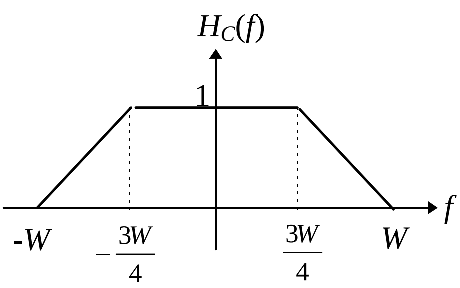
Consider the following communications system

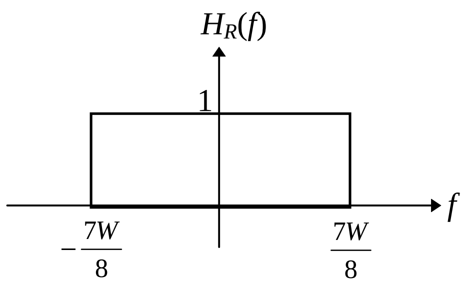


The transmitted signal is

$$X(t) = \sum_{m=-\infty}^{\infty} B_n v(t - mT)$$

where B_n is a random binary sequence of statistically independent ± 1. The transfer functions of the channel and the receiver filters are depicted in the figure below where $W = 5$ kHz.





The decision $\hat{B}_n$ is formed as the sign of $Y(nT)$, i.e., $\hat{B}_n = \mathrm{sgn}[Y(nT)]$

We say that the transmission is ISI free if $Y(nT) = kB_n$ for a constant $k > 0$ when $W(t) = 0$.

(a) What is the maximum symbol rate that can be achieved if $v(t)$ is a raised-cosine pulse with rolloff factor $\alpha = 0.25$ and if the transmission must be ISI free. (2p)

(b) What is the maximum symbol rate for an *arbitrary pulse* $v(t)$ if the transmission must take place without ISI? *Sketch* and *give* an expression for the Fourier transform $V(f)$ of such a pulse $v(t)$. (Ignore the fact that it might not be possible to implement $v(t)$ in a practical system.) (4p)

(c) Find the inverse Fourier transform $h_C(\tau) = F^{-1}\{H_C(f)\}$. (2p)

(d) Assume $H_C(f)$ is a matched filter output and find the impulse response and the transfer function of the matched filter. (2p)