2 ERT-algorithm in dimensioning a trunk

Dimension the trunk A-B for overload traffic using the ERT-method as shown in Figure 3. The allowed blocking rate B_o in all trunks is no more than 1%. The offered traffic for trunks and the trunk sizes are given in Table 1.

Table 1: Traffic offered and the link capacities

Connection	A_0	S
A–B	4	?
A–C	14	16
A–D	12	12
A–E	18	20

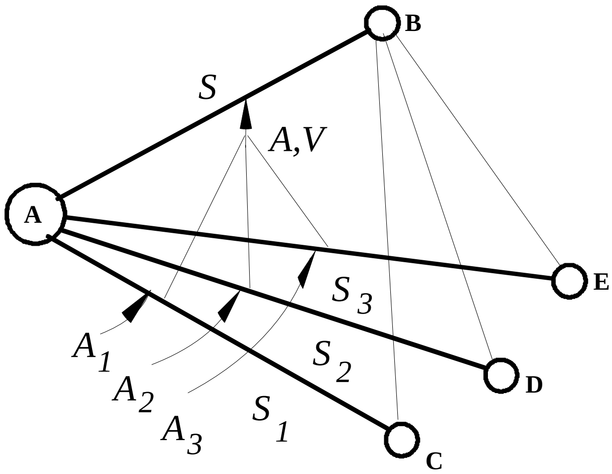

Figure 3: Trunk scheme

2.1 Equivalent Random Theory (ERT)

Kalevi Kilkki has written about the ERT-method as follows:

The determination of the overflow traffic is difficult in practise. For example, the overload trunk in Figure 3 is offered traffic from several other trunks.

In this kind of case one must aim to describe the characteristics of the traffic offered to the overload trunk. A simple and practical method is to determine the mean and the variance of the overload traffic. The mean of the overloaded traffic from each trunk is calculated with (1).

$$A_{f,a} = A_a B_E[A_a, S_a] \tag{1}$$

where the Erlang-B formula for blocked traffic[1]

where A_a is the offered traffic in Erlangs and S_a is the amount of service places is given in 3:

$$B_E[A_a, S_a] = \frac{\frac{{A_a}^{S_a}}{S_a!}}{\sum_{i=0}^{S_a} \frac{{A_a}^i}{i!}} \tag{3}$$

[1]The Erlang-B formula in recursive format (2) may come in handy in some environments:

$$\begin{aligned} B_e[0] &= 1 \\ B_e[A, S+1] &= \frac{1}{1 + \frac{S+1}{AB_e[A,S]}} = A\frac{B_e[A,S]}{(S+1) + AB_e[A,S]} \end{aligned} \tag{2}$$

For the variance of the overflown traffic:

$$V_{f,a} = A_{f,a}(1 - A_{f,a} + \frac{A_a}{S_a + 1 - A_a + A_{f,a}}) \tag{4}$$

And the mean of the traffic offered to the alternative overload trunk is summed by:

$$A_f = \sum A_{f,j} \tag{5}$$

and the variance is also summed from the individual varinces:

$$V_f = \sum V_{f,j} \tag{6}$$

The mean and the variance of the offered (overflown) traffic be calculated even in quite complicated cases. But how to determine the blocking rate of the overload trunk? One of the common methods is to use the ERT-method where when you know the mean and variance of the traffic one can build a system for the overflow traffic which has the previously determined parameters.

The following approximations are available ($z = V_f/A_f$):

$$A_i = \frac{(A_f + z - 1)^2 + \sqrt{A_f}(V_f + 3z(z-1))}{A_f + \sqrt{A_f}} \tag{7}$$

$$S_i = \frac{A_i(A_f + z)}{A_f + z - 1} - A_f - 1 \tag{8}$$

A_i and S_i can now be solved as A_f and V_f are known.

The blocked traffic is offered to the overload trunk and one can determine the blocking rate of the overload trunk o as follows:

$$B_o = \frac{B_E[A_i, S_i + S]}{B_E[A_i, S_i]} \tag{9}$$

When A_i and S_i are set the blocking rate of the overload trunk can be determined with (9). The problem is that the number of the service places is (usually) not an integer. The Erlang-B formula can be presented with gamma-function, so the amount of service places can be any real number.

A final note: What blocking is exactly described with the ERT-method? It was previously mentioned that the ERT-method gives the blocking rate of the overload trunk. However, we are usually interested in the blocking of the traffic, not the trunk. In general terms: The ERT-method gives us an estimate that predicts the average part of the call requests that get blocked when requesting access to the trunk. The values are aids when dimensioning an overload system, not accurate blocking rates.

Short ´´How To´´:

1. Determine the mean and the variance of the overflown traffic with (1), (3) and (4).
2. Sum up the means and variances of the overflown traffic as in (5) and in (6). Note. that the trunk already has some traffic (its mean and variance?).
3. Use your results to determine the traffic in the overload trunk (7) and the number of service places with (8).
4. Finally, determine the number of needed service places (9) with the given blocking rate.