

A Markovian Waypoint Mobility Model with Application to Hotspot Modeling

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Abstract—In this paper we introduce a Markovian random waypoint model which allows us to create diverse mobility patterns in the given movement domain. The model allows adjusting random pause times and the momentary velocity. Furthermore, the distribution used to pick the next waypoint may depend on the current location, which allows, e.g., creation of typical routes. As an application for the Markovian random waypoint we consider modelling hotspots. There are several mechanisms which may be causing a hotspot and we present how the parameters of the proposed model can be adjusted accordingly to match the scenario in question. We further illustrate how two seemingly similar mobility patterns lead to highly different estimates of achievable performance levels. Finally, we show how the state of the MWP model can be initialised in simulations such that the node starts from the stationary state without any initial transient.

INDEX TERMS: mobility modelling, random waypoint model, hotspots, Markov process in \mathbb{R}^2

I. INTRODUCTION

Mobility models can be largely categorised in two groups: (i) models that try to be realistic with respect to the movement and the topology of the area, and (ii) models that we call *elementary* in the sense that the models are simple and the mobility pattern does not reflect any realistic human movement. Whereas realistic mobility models may often be complex, the idea with elementary models is that their simplicity facilitates efficient simulations, while still capturing the essential impact of mobility on the performance issue under study.

One of the most widely used elementary mobility models in performance studies of MANETs is the random waypoint model (RWP), originally proposed in [1]. The properties of the RWP model have been analysed recently in a number of papers. Notably, the independence of the node speed distribution and the node location distribution has been discussed for example in [2] and [3] and rigorously shown in [4]. Accurate approximations for the stationary node distribution (i.e., the distribution for the location of a node) have been given in [2]. As part of our earlier work, we have derived an exact explicit result for the node distribution in [5]. Generalised

random mobility models have been analysed in [6] and [7] but, while providing insight, the results therein are not very explicit in terms of for example the node distribution.

The theoretical results can be utilised in many ways. Knowledge of the node distribution is needed in performance evaluation of MANETs, e.g., when analysing the connectivity properties with mobile nodes (see, e.g., [8]). Another important application is the use of the results to achieve so called perfect simulation (as defined in [6] and also studied in [3]), where the state of the mobility model is initialised so that the simulation can be started from the stationary state, without the need for any special transient handling. This is important from a practical point of view when performing simulation based performance studies with mobile nodes.

In this paper we present a Markovian Waypoint model (MWP), which is a generalised mobility model based on the basic RWP model. In the MWP model, mobile nodes move along a straight line segment from one waypoint to another at a certain speed. The waypoints are obtained from a discrete time Markov process having a subset of \mathbb{R}^2 as the state space. Additionally, we allow the velocity distribution of a node along a given leg (transition between two waypoints) to depend on the end points of the leg. Moreover, our model allows random pause times during the transitions in addition to the standard extension of allowing random pause times at the waypoints. However, the movement pattern of MWP still retains the characteristic that the direction of the movement is independent of the direction during the previous transition (or leg). This implies that, depending on the choice of the embedded Markov chain for the waypoints, the movement trajectory may contain sharp angles which can be considered unnatural. Hence, recent mobility models, such as [9] and [10] that are able to produce more smoothly turning trajectories may be more suitable models than MWP in this respect.

Our model can be seen as a generalisation of the model analysed in [7] by discretizing the continuous state space. In particular, in our model the velocity distribution is more general and the model also allows random pause times during the transitions. For this model, we provide general exact integral expressions for the mean transition time, mean velocity and the stationary node distribution.

We apply the MWP model for illustrating the various ways

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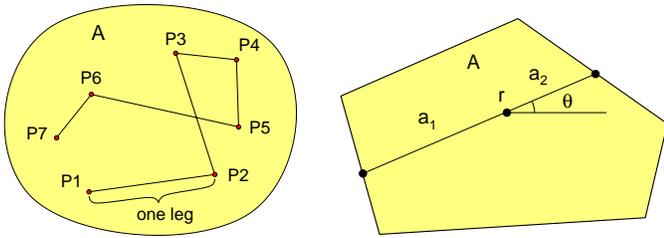


Fig. 1. Basic RWP process illustrated (left) and the notation in formula (1) (right).

of modeling hotspots from the point of view of network simulations. According to our definition, a hotspot is a part in the movement area, where the nodes spend a proportionally larger fraction of time, and this phenomenon can be achieved in several ways. This has been studied by using simulations in [11], but we rely on our exact results. We further compare the various ways of modeling the hotspots and illustrate through a simple example how the hotspot model (or the mechanism behind the hotspot phenomenon) may influence the obtained performance results. This serves as an argument for the necessity of having a rational reason for adopting a certain modeling approach. Additionally, we describe how the simulation of the MWP process can be started from the stationary distribution, which is referred to as perfect simulation.

The rest of the paper is organised as follows. In Section II we discuss the basic RWP model. The MWP model is formally defined in Section III, and then, in Section IV we present the relevant analytical results for the model. In Section V we discuss briefly the different realistic mechanisms behind hotspots and map them to the different model parameters, illustrated by examples. In Section VI we illustrate briefly how perfect simulation can be achieved for our model. Finally, Section VII contains the conclusions.

II. PRELIMINARIES AND THE BASIC RWP MODEL

The basic RWP process is defined by an infinite sequence,

$$(P_0, P_1), (P_1, P_2), (P_2, P_3), \dots$$

where each P_i is a so-called waypoint and (P_i, P_{i+1}) corresponds to one leg. The node simply moves with a constant velocity of v from one waypoint to another. A sample realisation is illustrated in Fig. 1 (left). The stationary node distribution of the basic RWP process is given by [5]

$$f(\mathbf{r}) = \frac{1}{\bar{\ell} \cdot A^2} \int_0^\pi a_1 a_2 (a_1 + a_2) d\theta, \quad (1)$$

where $\bar{\ell}$ is the mean length of a leg, A the area of the domain (assumed to be convex), $a_1 = a(\mathbf{r}, \theta + \pi)$ and $a_2 = a(\mathbf{r}, \theta)$ are the distances to the border from point \mathbf{r} in direction $\theta + \pi$ and θ , respectively (see Fig. 1 (right)). A similar integral formula can be also derived for the mean arrival rate into a given subset (see [12]).

As a simple extension to the basic RWP model one can introduce i.i.d. pause times at the waypoints before the node

continues on the next leg. The pdf of the node location in this case consists of two components corresponding to two modes (moving and paused) which alternate in the original process, see [2], [3].

III. MARKOVIAN WAYPOINT MODEL

In this section we give a definition for a more general model than RWP which we refer to as the Markovian waypoint model (MWP). The model is as follows.

Definition of the Markovian waypoint (MWP) model:

- 1) **Waypoints:** The node moves along a straight line segment from the current waypoint P_i to the next P_{i+1} . The waypoints P_0, P_1, P_2, \dots constitute a Markov chain in a convex area $\mathcal{A} \subset \mathbb{R}^2$ with transition probability densities given by $g(\mathbf{r}_1, \mathbf{r}_2) : \mathcal{A} \times \mathcal{A} \rightarrow \mathbb{R}$. Thus, given the current waypoint \mathbf{r}_1 , the next waypoint \mathbf{r}_2 is drawn using the conditional pdf $g(\mathbf{r}_1, \mathbf{r}_2)$.
- 2) **Velocity of the node:** Consider a leg $(\mathbf{r}_1, \mathbf{r}_2)$ and a point \mathbf{r} on it located at the distance of r from \mathbf{r}_1 , $\mathbf{r} = \mathbf{r}_1 + r \cdot \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$. We define the velocity for node on leg $(P_{i-1}, P_i) = (\mathbf{r}_1, \mathbf{r}_2)$ at point r as a conditional random function, $v_i(r) \sim \nu(r; \mathbf{r}_1, \mathbf{r}_2)$. Thus, for each leg i a random velocity $v_i(r)$ is drawn from $\nu(r; \mathbf{r}_1, \mathbf{r}_2)$, which defines the velocity along the leg i .
- 3) **Pause times at waypoints:** The node keeps a random pause time $\tau_i \sim \tau(P_i)$, at each waypoint P_i , where $\tau(\mathbf{r})$ is assumed to have a finite mean $\bar{\tau}(\mathbf{r}) < \infty \forall \mathbf{r} \in \mathcal{A}$.
- 4) **Poissonian pause times during transitions:** The node may also keep random pauses on each leg. Let $\gamma(r; \mathbf{r}_1, \mathbf{r}_2)$ denote the conditional intensity (per unit time, e.g., in 1/s) at which a moving node from \mathbf{r}_1 to \mathbf{r}_2 stops and goes to a pause state, and let the conditional random variable $S(r; \mathbf{r}_1, \mathbf{r}_2)$ denote the duration of the pause (a function of the location r on leg $(\mathbf{r}_1, \mathbf{r}_2)$).

The MWP model is a general model that allows a very versatile modeling of node movement. However, the MWP model can still be considered as an elementary mobility model in the sense that the path the node follows between two consecutive waypoints is a straight line.

The way the waypoints are chosen allows us to create, e.g., popular routes a user walks or drives through, i.e., the consecutive legs may be correlated. The velocity of the node, defined by the conditional random variables $\nu(r; \mathbf{r}_1, \mathbf{r}_2)$, allows us to model different kinds of transitions and terrain types. One can, e.g., define the velocity distribution based on the distance to be travelled, i.e., the longer transitions with speeds typical to a vehicular user, and shorter with speeds typical to a pedestrian user [7]. Or, $\nu(r; \mathbf{r}_1, \mathbf{r}_2)$ along a given leg may be defined such that it models the increase of speed from 0 to a random maximum speed (constant acceleration) and the subsequent deceleration. If the length of the leg is not enough, the target maximum speed will not be reached.

IV. ANALYTICAL RESULTS

In this section we will present several analytical results for the MWP process. First we will derive expressions for

the mean transition and pause times, the two modes which alternate in the complete process. Then, based on these we give an exact expression for the stationary node distribution.

Before proceeding, observe that in the MWP model the node may be in three different states at a given point of time: 1) the node is moving towards the next waypoint, 2) the node is keeping a pause on a leg towards the next waypoint, or 3) the node is keeping a pause at the current waypoint (see Fig. 2).

A. Mean transition and pause times

Let $\pi(\mathbf{r})$ denote the **stationary waypoint distribution**, i.e., the pdf of the location of an arbitrary waypoint, for which it holds that

$$\int_{\mathcal{A}} \pi(\mathbf{r}_1) \cdot g(\mathbf{r}_1, \mathbf{r}_2) d^2 \mathbf{r}_1 = \pi(\mathbf{r}_2), \text{ and} \quad (2)$$

$$\int_{\mathcal{A}} \pi(\mathbf{r}) d^2 \mathbf{r} = 1.$$

We assume that $\pi(\mathbf{r})$ exists and is well-defined. Let ℓ_i denote the length of leg i , $\ell_i = |P_i - P_{i-1}|$, and $\bar{\ell}$ the **mean length of a leg**,

$$\bar{\ell} = \int_{\mathcal{A}} d^2 \mathbf{r}_1 \pi(\mathbf{r}_1) \int_{\mathcal{A}} d^2 \mathbf{r}_2 g(\mathbf{r}_1, \mathbf{r}_2) |\mathbf{r}_2 - \mathbf{r}_1|.$$

We also assume that $\bar{\ell}$ is finite, which is the case if the diameter of \mathcal{A} is finite. Let T_i denote the **transition time of leg i** , i.e.,

$$T_i = \int_0^{\ell_i} \frac{dr}{v_i(r)}.$$

Note that the velocity distribution $v_i(r)$ is a random variable, $v_i(r) \sim \nu(r; P_{i-1}, P_i)$, and may depend on P_{i-1} and P_i .

Proposition 1 (mean transition time) *The mean transition time between two waypoints P_{i-1} and P_i , not including the pause time at the end of the leg, is given by*

$$\mathbb{E}[T] = \int_{\mathcal{A}} d^2 \mathbf{r}_1 \pi(\mathbf{r}_1) \int_{\mathcal{A}} d^2 \mathbf{r}_2 g(\mathbf{r}_1, \mathbf{r}_2) \int_0^{|\mathbf{r}_2 - \mathbf{r}_1|} dr \mu(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \mathbb{E}[\nu^{-1}(r; \mathbf{r}_1, \mathbf{r}_2)], \quad (3)$$

where

$$\mu(r; \mathbf{r}_1, \mathbf{r}_2) = 1 + \gamma(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \bar{S}(r; \mathbf{r}_1, \mathbf{r}_2).$$

Proof: Conditioning on the leg $(\mathbf{r}_1, \mathbf{r}_2)$ gives

$$\mathbb{E}[T] = \int_{\mathcal{A}} d^2 \mathbf{r}_1 \pi(\mathbf{r}_1) \int_{\mathcal{A}} d^2 \mathbf{r}_2 g(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathbb{E}[T|\mathbf{r}_1, \mathbf{r}_2].$$

Consider next an arbitrary leg $(\mathbf{r}_1, \mathbf{r}_2)$ going through \mathbf{r} ,

$$\mathbf{r} = \mathbf{r}_1 + r \cdot \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}, \quad (4)$$

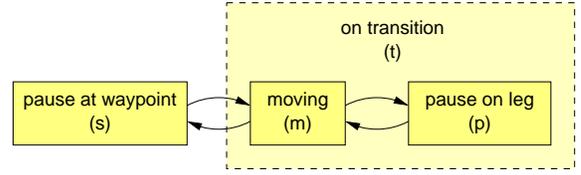


Fig. 2. Different modes of the MWP model.

and a short time interval of length Δt . Conditioned on a velocity distribution $v_i(r)$ along the leg i , without a possible stop the node travels a distance of $\Delta r = v_i(r) \cdot \Delta t$ during this time. However, with probability of $\gamma(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \Delta t$ a transition to pause mode occurs and it takes $\Delta t + S(r; \mathbf{r}_1, \mathbf{r}_2)$ time units from the node to travel the same distance of Δr . Hence, the mean time spent to travel the distance Δr is

$$\begin{aligned} \Delta T &= \Delta t + \Delta t \cdot \gamma(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \bar{S}(r; \mathbf{r}_1, \mathbf{r}_2) \\ &= \mu(r; \mathbf{r}_1, \mathbf{r}_2) \cdot v_i^{-1}(r; \mathbf{r}_1, \mathbf{r}_2) \Delta r \end{aligned}$$

and consequently,

$$\mathbb{E}[T|\mathbf{r}_1, \mathbf{r}_2] = \int_0^{|\mathbf{r}_2 - \mathbf{r}_1|} \mu(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \mathbb{E}[\nu^{-1}(r; \mathbf{r}_1, \mathbf{r}_2)]. \quad (5)$$

■

Corollary 2 (leg independent velocity) *If the velocity on each leg remains constant, the value of which is drawn when a new leg starts from a distribution which is independent on the end point locations of the leg, $\nu(r; \mathbf{r}_1, \mathbf{r}_2) \sim v$, and there are no pause times, then*

$$\mathbb{E}[T] = \bar{\ell} \cdot \mathbb{E}[1/v].$$

Proposition 3 (mean pause time) *The mean pause time before the next transition, denoted by $\mathbb{E}[\tau]$, is given by*

$$\mathbb{E}[\tau] = \int_{\mathcal{A}} d^2 \mathbf{r} \pi(\mathbf{r}) \bar{\tau}(\mathbf{r}). \quad (6)$$

The proof is trivial.

B. Stationary node distribution

The MWP process consists of two alternating states, transition and pause. Moreover, each transition consists of one or more moving periods separated by Poissonian pause times. Let $p^{(t)}$, $p^{(m)}$, $p^{(s)}$ and $p^{(p)}$ denote the fraction of time the node spends on transitions, the node is moving, the node is keeping a pause during a transition, and the node is keeping a pause at waypoints, respectively. For these we have,

$$\begin{aligned} p^{(t)} &= p^{(m)} + p^{(p)} = \frac{\mathbb{E}[T]}{\mathbb{E}[T] + \mathbb{E}[\tau]}, \text{ and} \\ p^{(s)} &= 1 - p^{(t)} = \frac{\mathbb{E}[\tau]}{\mathbb{E}[T] + \mathbb{E}[\tau]}. \end{aligned} \quad (7)$$

First consider a process without pause times at waypoints.

Proposition 4 (MWP without pause times at waypoints)

The stationary node distribution of the MWP process (g, ν, γ, S) is given by

$$f^{(t)}(\mathbf{r}) = \frac{1}{\mathbb{E}[T]} \int_0^{2\pi} d\phi \int_0^{a(\mathbf{r}, \phi + \pi)} dr_1 \pi(\mathbf{r}_1) \int_0^{a(\mathbf{r}, \phi)} dr_2 \quad (8)$$

$$(r_1 + r_2) \cdot g(\mathbf{r}_1, \mathbf{r}_2) \cdot \mu(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \mathbb{E}[\nu^{-1}(r_1; \mathbf{r}_1, \mathbf{r}_2)],$$

where

$$\mu(r; \mathbf{r}_1, \mathbf{r}_2) = 1 + \gamma(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \bar{S}(r; \mathbf{r}_1, \mathbf{r}_2),$$

and $a(\mathbf{r}, \phi)$ denotes the distance from point \mathbf{r} to the boundary $\partial\mathcal{A}$ in direction ϕ , $\mathbf{r}_1 = \mathbf{r} - r_1 \cdot (\cos \phi, \sin \phi)$ and $\mathbf{r}_2 = \mathbf{r} + r_2 \cdot (\cos \phi, \sin \phi)$.

Proof: We consider the mobile component, i.e., a process from which pause times at the waypoints have been removed ($\tau(\mathbf{r}) = 0 \forall \mathbf{r}$). The probability of finding a node inside a small area element dA around point $\mathbf{r} \in \mathcal{A}$ at an arbitrary point of time is clearly equal to the time the node spends inside dA during an arbitrary leg multiplied by the leg generation rate (cf. Little's result). The legs are generated at the rate of

$$\frac{1}{\mathbb{E}[T]}.$$

Let $\mathbb{E}[\Delta t]$ denote the mean time spent inside dA during an arbitrary leg. Thus the pdf of the stationary node location is

$$f^{(t)}(\mathbf{r}) = \frac{\mathbb{E}[\Delta t]}{\mathbb{E}[T] \cdot dA}.$$

Our approach is based on conditioning on the leg and the velocity distribution, $(P_{i-1}, P_i, v_i(r))$, for which we have

$$\mathbb{E}[\Delta t] = \mathbb{E}[\mathbb{E}[\Delta t | P_{i-1} = \mathbf{r}_1, P_i = \mathbf{r}_2, v_i(r) = v(r)]].$$

From (2) we can obtain $\pi(\mathbf{r})$, which corresponds to the pdf that an arbitrary leg starts from \mathbf{r} . Conditioning on the starting point of the leg $P_{i-1} = \mathbf{r}_1$ gives

$$\begin{aligned} f^{(t)}(\mathbf{r}) &= \int_{\mathcal{A}} \pi(\mathbf{r}_1) \cdot \mathbb{E}[\Delta t | \mathbf{r}_1] d^2\mathbf{r}_1 \\ &= \int_0^{2\pi} \int_0^{a(\mathbf{r}, \phi + \pi)} r_1 \cdot \pi(\mathbf{r}_1) \cdot \mathbb{E}[\Delta t | \mathbf{r}_1] dr_1 d\phi, \quad (9) \end{aligned}$$

where in the latter form $\mathbf{r}_1 = \mathbf{r} - r_1 \cdot (\cos \phi, \sin \phi)$ and $a(\mathbf{r}, \phi)$ is the distance from point \mathbf{r} to the boundary in direction ϕ .

Next we condition on the end point $P_i = \mathbf{r}_2$. According to Fig. 3 the possible destination area of legs going through dA at the distance of $r_1 + r_2$ is equal to $(r_1 + r_2) d\theta dr_2$. Thus,

$$\mathbb{E}[\Delta t | \mathbf{r}_1] = \int_0^{a(\mathbf{r}, \phi)} \mathbb{E}[\Delta t | \mathbf{r}_1, \mathbf{r}_2] \cdot (r_1 + r_2) \cdot g(\mathbf{r}_1, \mathbf{r}_2) d\phi dr_2. \quad (10)$$

Finally, we condition on the velocity distribution on leg $(\mathbf{r}_1, \mathbf{r}_2)$ denoted by $v_i(r)$. In particular, we condition on that the velocity of the node at point \mathbf{r} is $v_i(r_1) = v(r_1)$. We

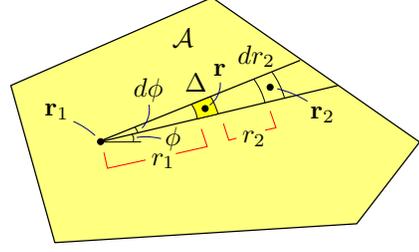


Fig. 3. Illustration of derivation.

must also explicitly take into account a possible Poissonian pause within dA . Let Δ denote the length of the line segment $(\mathbf{r}_1, \mathbf{r}_2)$ residing inside dA (see Fig. 3). If the node does not pause during a transition through dA the time spent in dA is

$$\Delta t_0 = \frac{\Delta}{v(r_1)},$$

which happens with the probability of $1 - \gamma(r_1; \mathbf{r}_1, \mathbf{r}_2) \cdot \Delta t_0$. Similarly, with a probability of $\gamma(r_1; \mathbf{r}_1, \mathbf{r}_2) \cdot \Delta t_0$ a Poissonian pause time occurs and the time spent inside dA is given by

$$\Delta t_1 = \frac{\Delta}{v(r_1)} + S(r_1; \mathbf{r}_1, \mathbf{r}_2),$$

and the mean sojourn time in dA on leg $(\mathbf{r}_1, \mathbf{r}_2)$ with $v(r)$ is

$$\begin{aligned} \mathbb{E}[\Delta t | P_{i-1} = \mathbf{r}_1, P_i = \mathbf{r}_2, v_i(r) = v(r)] \\ &= \frac{\Delta}{v(r_1)} \cdot (1 + \gamma(r_1; \mathbf{r}_1, \mathbf{r}_2) \cdot \bar{S}(r_1; \mathbf{r}_1, \mathbf{r}_2)) \\ &= \frac{\Delta \cdot \mu(r_1; \mathbf{r}_1, \mathbf{r}_2)}{v(r_1)}, \end{aligned}$$

which yields,

$$\begin{aligned} \mathbb{E}[\Delta t | P_{i-1} = \mathbf{r}_1, P_i = \mathbf{r}_2] \\ &= \Delta \cdot \mu(r_1; \mathbf{r}_1, \mathbf{r}_2) \cdot \mathbb{E}[\nu^{-1}(r_1; \mathbf{r}_1, \mathbf{r}_2)]. \quad (11) \end{aligned}$$

Next we refer again to Fig. 3 and note that $dA = \Delta \cdot r_1 d\phi$. Substituting this into (11) gives

$$\begin{aligned} \mathbb{E}[\Delta t | P_{i-1} = \mathbf{r}_1, P_i = \mathbf{r}_2] \\ &= \frac{dA}{d\phi r_1} \cdot \mu(r_1; \mathbf{r}_1, \mathbf{r}_2) \cdot \mathbb{E}[\nu^{-1}(r_1; \mathbf{r}_1, \mathbf{r}_2)]. \end{aligned}$$

Combining this with (9) and (10) completes the proof. ■

It turns out, as can be seen from the previous proof, that from the point of view of the stationary distribution, the Poissonian pause times corresponding to state 2) can be replaced by adjusting the velocity ν in an appropriate way:

Corollary 5 (Poissonian pause times and (mean) velocity)

The MWP process $(g, \nu, \tau, \gamma, S)$ with the Poissonian pause times, defined by $\gamma(r; \mathbf{r}_1, \mathbf{r}_2)$ and $S(r; \mathbf{r}_1, \mathbf{r}_2)$, has the same stationary node distribution as a MWP process (g, ν^*, τ) without Poissonian pause times when

$$\nu^*(r; \mathbf{r}_1, \mathbf{r}_2) = \frac{\nu(r; \mathbf{r}_1, \mathbf{r}_2)}{\mu(r; \mathbf{r}_1, \mathbf{r}_2)} = \frac{\nu(r; \mathbf{r}_1, \mathbf{r}_2)}{1 + \gamma(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \bar{S}(r; \mathbf{r}_1, \mathbf{r}_2)}.$$

Proposition 6 (pause times at waypoints) *The pdf of the node location on condition that the node is in pause state at a waypoint is given by*

$$f^{(s)}(\mathbf{r}) = \frac{\pi(\mathbf{r}) \cdot \bar{\tau}(\mathbf{r})}{\mathbb{E}[\tau]}. \quad (12)$$

Proof: Consider a differential area element dA about \mathbf{r} . The probability that an arbitrary waypoint is in dA is equal to $\pi(\mathbf{r}) dA$, and the mean pause time in dA , according to (6), is equal to $\bar{\tau}$. Thus, the time spent in dA in pause mode is proportional to $\pi(\mathbf{r}) \cdot \bar{\tau}$. $\mathbb{E}[\tau]$ is a normalisation constant. ■

Corollary 7 (complete process) *The pdf of the node location of the complete process can be written as a sum*

$$f(\mathbf{r}) = p^{(t)} \cdot f^{(t)}(\mathbf{r}) + p^{(s)} \cdot f^{(s)}(\mathbf{r}). \quad (13)$$

Corollary 8 (mean velocity) *The mean velocity of the node, denoted by $\mathbb{E}[v]$, is given by*

$$\mathbb{E}[v] = p^{(t)} \cdot \frac{\mathbb{E}[T]}{\bar{\ell}}.$$

If one is interested in further distinguishing between the actual movement and the pause times during the transitions, the respective time proportions and conditional pdf's are easy to determine. In particular, consider any long realisation consisting of moving periods (m), Poissonian pause times during transitions (p), and pause times at waypoints (s). Let the respective time periods in different modes be T_m , T_p and T_s , for which we have

$$\frac{T_m + T_p}{T_m + T_p + T_s} \rightarrow p^{(t)}.$$

Consider next a process from which the pause times during the transitions have been removed, i.e., the time periods contributing to T_p . The two realisations, the original and the reduced, are always equally likely to occur in the respective process. For the reduced realisation we obtain similarly that

$$\frac{T_m}{T_m + T_s} \rightarrow p^{(t*)}.$$

Combining the above relations we have

$$\frac{T_m}{T_m + T_p} \rightarrow \frac{p^{(t*)}(1 - p^{(t)})}{p^{(t)}(1 - p^{(t*)})}.$$

Using (7) we can determine both $p^{(t)}$ and $p^{(t*)}$, and

$$p^{(m)} = \frac{p^{(t*)}(1 - p^{(t)})}{1 - p^{(t*)}}, \quad \text{and} \quad p^{(p)} = \frac{p^{(t)} - p^{(t*)}}{1 - p^{(t*)}}.$$

The conditional pdf of the mobile component can be obtained by considering a modified model where $\gamma(\mathbf{r}) = 0 \forall \mathbf{r}$, i.e.

$$f^{(m)}(\mathbf{r}) = f_{|\gamma=0}^{(t)}(\mathbf{r}). \quad (14)$$

Moreover, the conditional pdf of the Poissonian pause times during the transitions is given by

$$f^{(p)}(\mathbf{r}) = \frac{1}{p^{(p)}} \cdot \left(p^{(t)} f^{(t)}(\mathbf{r}) - p^{(m)} f^{(m)}(\mathbf{r}) \right). \quad (15)$$

Finally, a decomposition of the pdf of the node location is

$$f(\mathbf{r}) = p^{(m)} f^{(m)}(\mathbf{r}) + p^{(p)} f^{(p)}(\mathbf{r}) + p^{(s)} f^{(s)}(\mathbf{r}). \quad (16)$$

C. Examples of Special Cases

1) Velocity independent from the waypoints: Assume the velocity distribution ν depends only on the current location \mathbf{r} , not the starting or end points of the leg, i.e., $\nu = \nu(\mathbf{r})$, and that there are no Poissonian pause times (or they depend also only on the current location and have been incorporated according to Corollary 5). Then, (8) can be written as

$$f^{(t)}(\mathbf{r}) = \frac{\mathbb{E}[\nu^{-1}(\mathbf{r})]}{\mathbb{E}[T]} \int_0^{a(\mathbf{r}, \phi + \pi)} \int_0^{2\pi} \pi(\mathbf{r}_1) \int_0^{a(\mathbf{r}, \phi)} (r_1 + r_2) \cdot g(\mathbf{r}_1, \mathbf{r}_2) dr_2 dr_1 d\phi.$$

2) Independent and Identically Distributed Waypoints: Assume that the waypoints P_i are i.i.d. random variables having a common pdf $g(\mathbf{r})$, and that the velocity distribution is independent of the waypoints, $\nu = \nu(\mathbf{r})$. In this case, $g(\mathbf{r}_1, \mathbf{r}_2) = g(\mathbf{r}_2)$ and $\pi(\mathbf{r}) = g(\mathbf{r})$. Consequently, the transition component of the pdf of the node location, given by (8), can be written as (symmetry)

$$f^{(t)}(\mathbf{r}) = \frac{2 \mathbb{E}[\nu^{-1}(\mathbf{r})]}{\mathbb{E}[T]} \int_0^{2\pi} d\phi \left[\int_0^{a(\mathbf{r}, \phi + \pi)} dr_1 r_1 \cdot g(\mathbf{r}_1) \cdot \int_0^{a(\mathbf{r}, \phi)} dr_2 g(\mathbf{r}_2) \right].$$

3) Uniform Waypoint Distribution: Denote by A the area of the domain, $A = |\mathcal{A}|$, and assume uniformly distributed waypoints on \mathcal{A} , $g(\mathbf{r}) = 1/A$, and that the velocity distribution is independent of the waypoints, $\nu = \nu(\mathbf{r})$. In this case, the mobile component of the pdf of the node location reduces to

$$f^{(t)}(\mathbf{r}) = \frac{\mathbb{E}[\nu^{-1}(\mathbf{r})]}{\mathbb{E}[T] A^2} \int_0^\pi a_1 a_2 (a_1 + a_2) d\phi,$$

where $a_1 = a(\mathbf{r}, \phi + \pi)$ and $a_2 = a(\mathbf{r}, \phi)$, see Fig. 1 (right). Similarly, the mean pause time is given by

$$\mathbb{E}[\tau] = \frac{1}{A} \int_{\mathcal{A}} d^2 \mathbf{r} \bar{\tau}(\mathbf{r}).$$

If we further assume a constant spatial velocity $\nu(\mathbf{r}) = v \forall \mathbf{r} \in \mathcal{A}$ and zero pause times at the waypoints, $\tau(\mathbf{r}) = 0 \forall \mathbf{r} \in \mathcal{A}$, then the relation between the mean leg length $\bar{\ell}$ and the mean transition time $\mathbb{E}[T] = \bar{\ell}/v$, and we obtain the basic RWP process. In this case the pdf for the node distribution reduces to (1).

V. MODELLING HOTSPOTS WITH MWP

As an example application of the MWP model we consider the problem of modelling hotspots. In this section we limit ourselves to models where the spatial velocity component and the Poissonian pause times are independent of the waypoints, i.e., $\nu = \nu(\mathbf{r})$, $\gamma = \gamma(\mathbf{r})$ and $S = S(\mathbf{r})$.

A. Mechanisms behind a Hotspot

By a hotspot we refer to a situation where a considerable number of users reside within a certain area. Consider some area which we are interested in. This area can be perceived as a black box where users arrive. According to the Little's formula $\bar{N} = \lambda \cdot \bar{D}$, where \bar{N} denotes the mean number of users in the area, λ the user arrival rate, and \bar{D} the mean sojourn time of a user. The area is a hotspot if \bar{N} is considerably higher than elsewhere. Hence, either λ or \bar{D} (or both) must be higher than elsewhere in order to form a hotspot.

The actual reasons leading to a hotspot can be numerous. From the perspective of the model we want to know the mechanisms giving rise to a higher node density. For example, at 8 am (or 4 pm) people might be travelling to work (or home) and the underground stops might become hotspots. Generally, this kind of a scenario results from a location being a popular connecting area in a public transport system. Thus, a hotspot may be a result of users visiting often a certain area. Alternatively, it may develop as a result of users spending a considerable amount of time in the area. Examples of this kind of a scenario are an open air concert and a restaurant area in the evening.

B. Modelling Alternatives

Next we discuss how the properties of the MWP process can be used to model the hotspot scenarios described above. At least, the following approaches are possible:

- 1) **Spatial pause times** at waypoints that allow the node to keep a longer pause in the hotspot area. A standard extension to the basic model is to introduce independent pause times at the waypoints before the node proceeds on the next leg. The pause times can also depend on the current location.
- 2) **Poisson pause times** during transitions that allow the node to stop in the hotspot area for a random time with rate $\gamma(\mathbf{r})$ per time unit. This results in a different spatial distribution for the locations of the pauses than with spatial pause times at waypoints. The equivalent spatial (mean) velocity $\nu^*(\mathbf{r})$ is given by (4) and the stationary node distribution is straightforward to determine.
- 3) **Spatial velocity** that makes the node move slower within the hotspot area. The spatial velocity allows a very general formulation for the node velocity that can include random elements, as well as location dependent features. The pdf resulting from the model with spatial velocity component $\nu(\mathbf{r})$ is proportional to $f^*(\mathbf{r}) \mathbb{E}[\nu^{-1}(\mathbf{r})]$, where $f^*(\mathbf{r})$ is the pdf of the basic RWP model.
- 4) **Non-uniform waypoint distribution** that makes the node visit the hotspot area more often. The stationary node distribution with a non-uniform waypoint distribution can still be computed (numerically at least), even though it increases the number of nested integrals in the expression. This corresponds to, e.g., hotspots at the connecting areas of transport systems.

C. Examples of Hotspot Scenarios

Before going into the examples let us introduce some auxiliary notation. The hotspot area is denoted by \mathcal{H} and its area by $H = |\mathcal{H}|$. Also, recall that $f^*(\mathbf{r})$ denotes the pdf of the node location of the basic RWP process, as given by (1), where a node moves at a constant velocity and keeps no pause times. The proportion of time the node in the basic RWP process spends in the hotspot area is denoted by p_H ,

$$p_H = \int_{\mathcal{H}} f^*(\mathbf{r}) d^2\mathbf{r}.$$

1) *Pause Times at Waypoints:* Consider first the case where the waypoints are uniformly distributed, $g(\mathbf{r}) = 1/A$, spatial velocity is constant, $\nu(\mathbf{r}) = 1$, no pauses occur during the transitions, $\gamma(\mathbf{r}) = 0$, and the pause times at waypoints are

$$\tau(\mathbf{r}) = \mathbf{1}_{\mathbf{r} \in \mathcal{H}} \cdot S,$$

where S is an i.i.d. random variable and \mathcal{H} is some subset of \mathcal{A} corresponding to a hotspot area. This corresponds to the basic RWP process with pause times at waypoints. In this case, the mean pause time is

$$\mathbb{E}[\tau] = \frac{H \cdot \bar{S}}{A} \quad \text{and} \quad f^{(s)}(\mathbf{r}) = \frac{\mathbf{1}_{\mathbf{r} \in \mathcal{H}}}{H}.$$

Consequently,

$$f(\mathbf{r}) = q \cdot f^*(\mathbf{r}) + \frac{1-q}{H} \cdot \mathbf{1}_{\mathbf{r} \in \mathcal{H}},$$

where

$$q = p^{(t)} = \frac{\bar{\ell} \cdot \mathbb{E}[1/v]}{\bar{\ell} \cdot \mathbb{E}[1/v] + H \cdot \bar{S}/A}.$$

Note that the parameter q represents the proportion of time the node is moving instead of keeping a pause at a waypoint. Consequently, the fraction of the user population residing in the hotspot area on average is given by

$$z_H = 1 - q + q \cdot p_H.$$

Let us furthermore assume that the domain \mathcal{A} is a unit disk and the hotspot area \mathcal{H} is a concentric disk with radius h , $h < 1$, as illustrated in Fig. 4. In this case, the cdf of the node's distance from the origin is

$$F_R(r) = q \cdot F_R^{(*)}(r) + (1-q) \cdot \left(\frac{\min\{r, h\}}{h} \right)^2,$$

where $F_R^{(*)}(r)$ is the cdf of the node distance from the origin in a unit disk for the basic RWP model (see [5]),

$$F_R^{(*)}(r) = \int_0^r \frac{45x(1-x^2)}{32} \int_0^\pi \sqrt{1-x^2 \cos^2 \phi} d\phi dx. \quad (17)$$

For $r \geq h$ the above reduces into

$$F_R(r) = 1 - q \cdot G_R^{(*)}(r),$$

where $G_R^{(*)}(r) = 1 - F_R^{(*)}(r)$. Fig. 5 (left) illustrates a case with $q = 0.8$ and $h = 0.3$. We note that the node density inside the hotspot is at least twice as large as elsewhere.

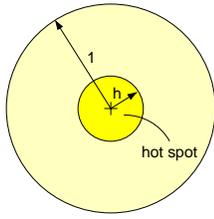


Fig. 4. Hotspot with radius of h in the center of unit disk.

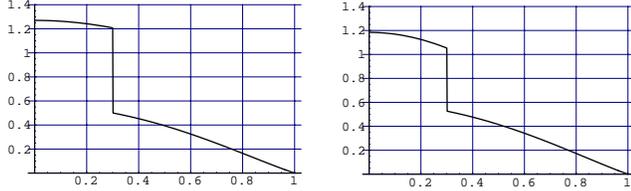


Fig. 5. Resulting pdfs of the node location in a unit disk with a hotspot at the center having a radius of 0.3. In left figure the hotspot is due to the additional pause times with parameters chosen such that the fraction of time the nodes are moving is 80%. In right figure a similar hotspot is created by a slower motion in the hotspot area, i.e., the velocity of the nodes in the hotspot area is 50% of the normal velocity.

2) *Slower Movement at Hotspot*: Let us next consider another mechanism behind the hotspot, i.e., a decreased velocity at which the users move inside the hotspot. In particular, we assume that the velocity outside the hotspot is q times greater than inside the hotspot. In this case it turns out that the resulting pdf of the node location is given by

$$f(\mathbf{r}) = \frac{1}{1 + (q-1)p_H} \cdot (1 + (q-1) \cdot 1_{\mathbf{r} \in \mathcal{H}}) \cdot f^*(\mathbf{r}).$$

In the case of a unit disk, where the hotspot is a smaller disk located in the center, we obtain

$$f(r) = \frac{1}{1 + (q-1)F_R^{(*)}(h)} \cdot (1 + (q-1) \cdot 1_{r < h}) \cdot f^*(r),$$

where $F_R^{(*)}(h)$ is given by (17). Furthermore, the fraction of users located in the hotspot area on average is given by

$$z_H = \frac{q \cdot p_H}{1 + (q-1) \cdot p_H}.$$

Again, in Fig. 5 (right) one example case is illustrated where $q = 2$ and $h = 0.3$. The two pdf's (left and right figure) are very similar with the chosen parameter values.

D. Flow Level Performance

As a final example let us consider a 3G cellular network where adaptive link rates depending on the signal quality are in use. Let random variable R denote the feasible link rate at the current location of the node. Assuming the nodes are still or move very slowly when transferring files, it can be shown that the capacity of the cell is given by (example adopted from [13], [14])

$$C = E[1/R]^{-1},$$

based on the model of single server PS-queues.

TABLE I
BASE STATION AND FOUR CAPACITY ZONES AROUND IT.

zone	distance	link rate R_i
1	1 km	10 Mbit/s
2	2 km	5 Mbit/s
3	3 km	2 Mbit/s
4	4 km	1 Mbit/s

Let us assume that the cell corresponds to some hotspot where numerous users move according to MWP process and download documents only when they have stopped. As it was mentioned previously, the pause times can be generated in two ways: 1) keeping a pause at the waypoints, or 2) keeping random pause times during the transitions. In particular, we consider two cases:

- i.i.d. pause times at waypoints
- i.i.d. pause times during the transitions

Otherwise, we assume the basic model with uniform waypoint distribution. It turns out that these two seemingly quite similar mobility models lead to considerably different performances in terms of cell capacity. In a) the locations where the download requests arrive are clearly uniformly distributed as the waypoints were uniformly distributed. On the other hand, in the model where a node keeps the pause times during the transitions the download requests originate according to stationary node distribution (1) of the basic RWP process.

For simplicity, let us furthermore assume that the cell has a maximum range of 4 km and that the feasible (unshared) link rate depends solely on the distance according to Table I. The proportion of download requests arriving from each zone in both cases, denoted by $\mathbf{P}^{(a)}$ and $\mathbf{P}^{(b)}$, respectively, are

$$\mathbf{P}^{(a)} = (1/16, 3/16, 5/16, 7/16)$$

$$\mathbf{P}^{(b)} = (0.13, 0.34, 0.37, 0.17)$$

Consequently, the cell capacities, given by

$$C = E[1/R]^{-1} = \left(\sum_i P_i \cdot (1/R_i) \right)^{-1}$$

are

$$C^{(a)} = 80/51 \approx 1.57 \text{ Mbit/s},$$

$$C^{(b)} \approx 2.33 \text{ Mbit/s}.$$

Thus, depending on how the stopping times are modelled, the resulting performance measure, the cell capacity, can vary a lot. At the capacity limit the zone specific loads are

$$\boldsymbol{\rho}^{(a)} = (0.01, 0.06, 0.25, 0.69),$$

$$\boldsymbol{\rho}^{(b)} = (0.03, 0.16, 0.43, 0.39).$$

i.e., a significant proportion (69%) of the load originates from the outermost zone in case a), causing the degradation of the performance.

VI. PERFECT SIMULATION OF MWP

Perfect simulation refers to the initialisation of the state of the system in a discrete event simulation so that it corresponds to the stationary state of the model. This has been addressed in numerous papers (see, e.g., [6] and [3]), and here we only briefly indicate how the perfect simulation can be achieved for the MWP model. Our approach is based on the rejection method, i.e., we generate samples and accept them with a certain probability. The pdf for a leg $\mathbf{r}_1 \rightarrow \mathbf{r}_2$ is

$$\pi(\mathbf{r}_1) \cdot g(\mathbf{r}_1, \mathbf{r}_2).$$

Conditioning on leg $\mathbf{r}_1 \rightarrow \mathbf{r}_2$, the mean time spent in a small interval of length dr on the leg consists of two components (moving and pause), as illustrated in Fig. 6. Define

$$G_1 = \max_{\mathbf{r}_1, \mathbf{r}_2} \pi(\mathbf{r}_1) \cdot g(\mathbf{r}_1, \mathbf{r}_2),$$

which is assumed to be finite. Furthermore, the conditional mean transition time is given by (5). Define

$$\begin{aligned} G_2 &= \max_{\mathbf{r}_1, \mathbf{r}_2, v(r)} \mathbb{E}[T | \mathbf{r}_1, \mathbf{r}_2] \\ &= \max_{\mathbf{r}_1, \mathbf{r}_2, v(r)} \int_0^\ell \mu(r; \mathbf{r}_1, \mathbf{r}_2) \cdot v^{-1}(r) dr, \end{aligned}$$

which we assume to be finite. The initialisation according to the stationary distribution can be performed as follows:

- 1) With probability of $p^{(s)}$: choose waypoint \mathbf{r}_0 using pdf

$$\pi(\mathbf{r}) \cdot \bar{\tau}(\mathbf{r}) / \mathbb{E}[\tau],$$

and the pause time t_0 using the residual pause time pdf.

return: pause time t_0 at waypoint \mathbf{r}_0 .

- 2) Draw $(\mathbf{r}_1, \mathbf{r}_2)$ using uniform distribution on $\mathcal{A} \times \mathcal{A}$.
- 3) Accept $(\mathbf{r}_1, \mathbf{r}_2)$ with probability of $\pi(\mathbf{r}_1) \cdot g(\mathbf{r}_1, \mathbf{r}_2) / G_1$, otherwise go back to step 2.
- 4) Draw random velocity for the leg, $v(r) \sim \nu(r; \mathbf{r}_1, \mathbf{r}_2)$.
- 5) Compute the transition and pause time components,

$$T_0 = \int_0^\ell \frac{dr}{v(r)}, \quad T_1 = \int_0^\ell \gamma(r; \mathbf{r}_1, \mathbf{r}_2) \frac{\bar{S}(r; \mathbf{r}_1, \mathbf{r}_2)}{v(r)} dr.$$

- 6) Accept velocity $v(r)$ with probability of $(T_0 + T_1) / G_2$, otherwise go back to step 4.
- 7) With probability of $\frac{T_0}{T_0 + T_1}$, draw r_0 with pdf $\propto 1/v(r)$. **return:** moving at r_0 on leg $(\mathbf{r}_1, \mathbf{r}_2)$ with $v(r)$.
- 8) Draw r_0 with pdf $\gamma(r; \mathbf{r}_1, \mathbf{r}_2) \cdot \bar{S}(r; \mathbf{r}_1, \mathbf{r}_2) / v(r)$, and pause time, t_0 , using the residual pdf of $S(r; \mathbf{r}_1, \mathbf{r}_2)$. **return:** pause for t_0 at r_0 , and then continue on leg $(\mathbf{r}_1, \mathbf{r}_2)$ with velocity $v(r)$.

VII. CONCLUSIONS

In this paper, we have introduced the MWP model, which is a general mobility model that allows a Markovian structure for the velocities, pause times and waypoints of the user. Several analytical results for this model were derived, notably an explicit integral formula for the stationary node distribution.

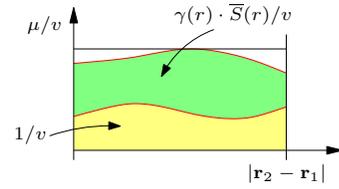


Fig. 6. Averaged sample path realization for a leg $\mathbf{r}_1 \rightarrow \mathbf{r}_2$. The (mean) time spent in each small interval of length dr is a sum of two components.

The model was adapted for modeling hotspots in several different ways. With a numerical example it was shown how one can achieve very similar mobility patterns in terms of the stationary node distribution using these different hotspot creation alternatives. It was also illustrated how the appropriate parameter values can be determined for creating a desired hotspot at a given location. However, in another example, it was demonstrated how seemingly very similar mobility models lead to highly different performances, emphasising the need for sound judgement when determining which modelling alternative to use. Finally, an algorithm was given for starting a simulation of the MWP model from the stationary distribution avoiding the need for special transient handling.

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