



# Flow model for a TCP/SIMA-like system with two NBR-classes

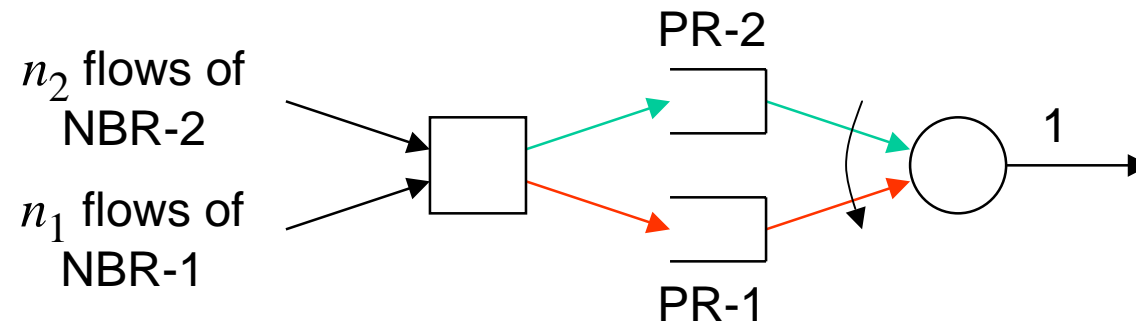
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# Contents

- Two priority classes
- Three priority classes

## Flow model for two priority classes



	NBR-1	NBR-2
PR-2	$ABR < b$	$ABR < \phi b$
	$b$	$\phi b$
PR-1	$ABR > b$	$ABR > \phi b$

## Equations for actual bit rates

$n_{ij} :=$  nr of flows of NBR- $i$  and PR- $j$   
 $\beta_{ij} :=$  ABR of flows of NBR- $i$  and PR- $j$

$$\beta_{12} = \min\left\{\frac{1}{n_{12}+n_{22}}, b\right\}$$

$$\beta_{22} = \min\left\{\max\left\{\frac{1}{n_{12}+n_{22}}, \frac{1-n_{12}b}{n_{22}}\right\}, \phi b\right\}$$

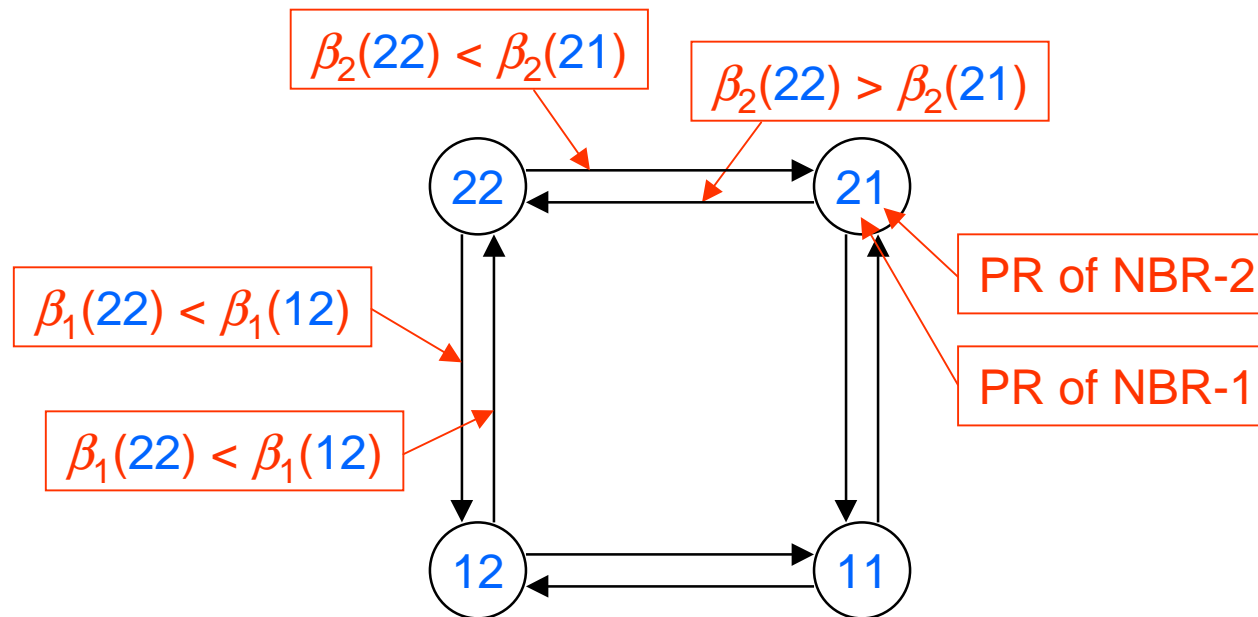
$$\beta_{11} = \max\left\{\frac{1-n_{12}\beta_{12}-n_{22}\beta_{22}}{n_{11}+n_{21}}, 0\right\}$$

$$\beta_{21} = \beta_{11}$$

NBR

PR

## Individually optimal actual bit rates (1)

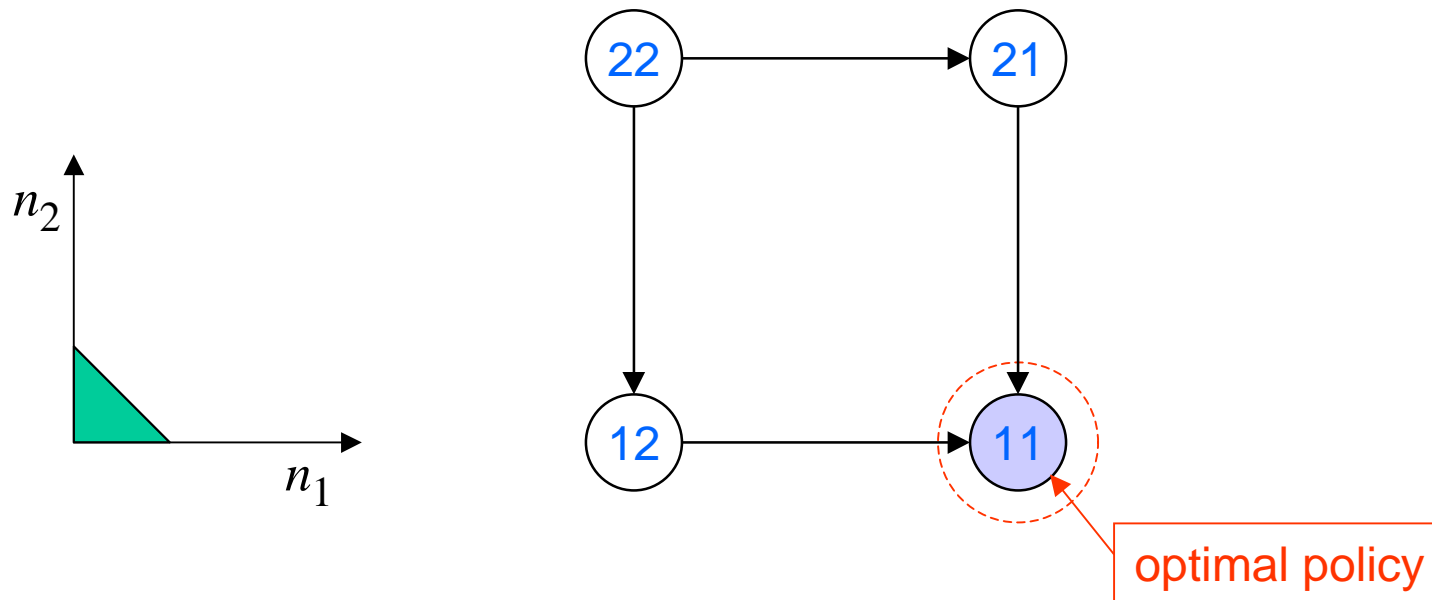


$$\beta_2(22) := \beta_{22}(n_{11}=0, n_{12}=n_1, n_{21}=0, n_{22}=n_2)$$

$$\beta_2(21) := \beta_{21}(n_{11}=0, n_{12}=n_1, n_{21}=n_2, n_{22}=0)$$

## Individually optimal actual bit rates (2)

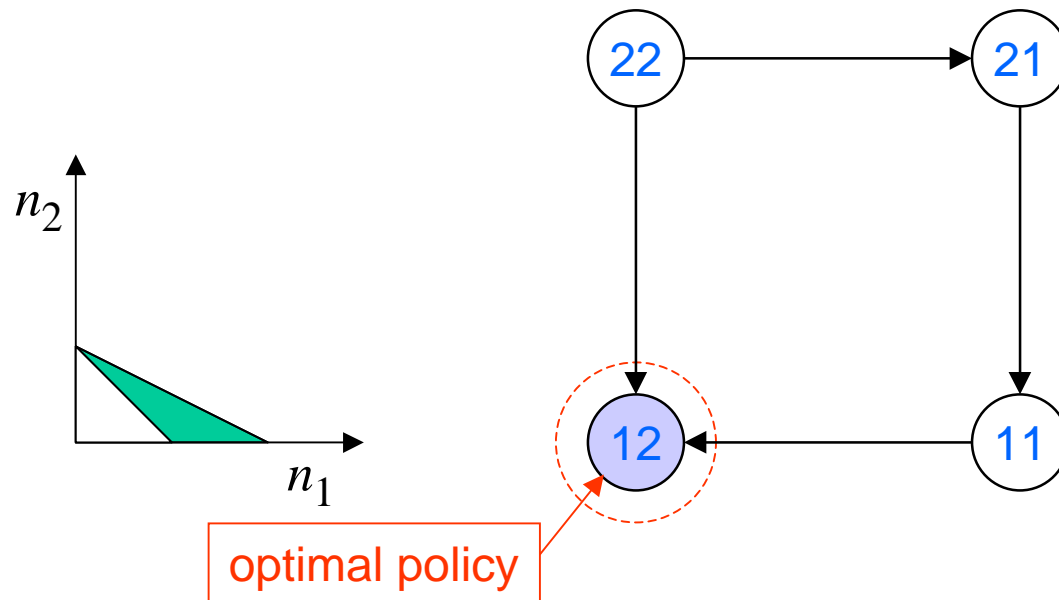
$$(A) n_1 \phi b + n_2 \phi b < 1$$



## Individually optimal actual bit rates (3)

$$(A^c) \quad n_1 \phi b + n_2 \phi b > 1$$

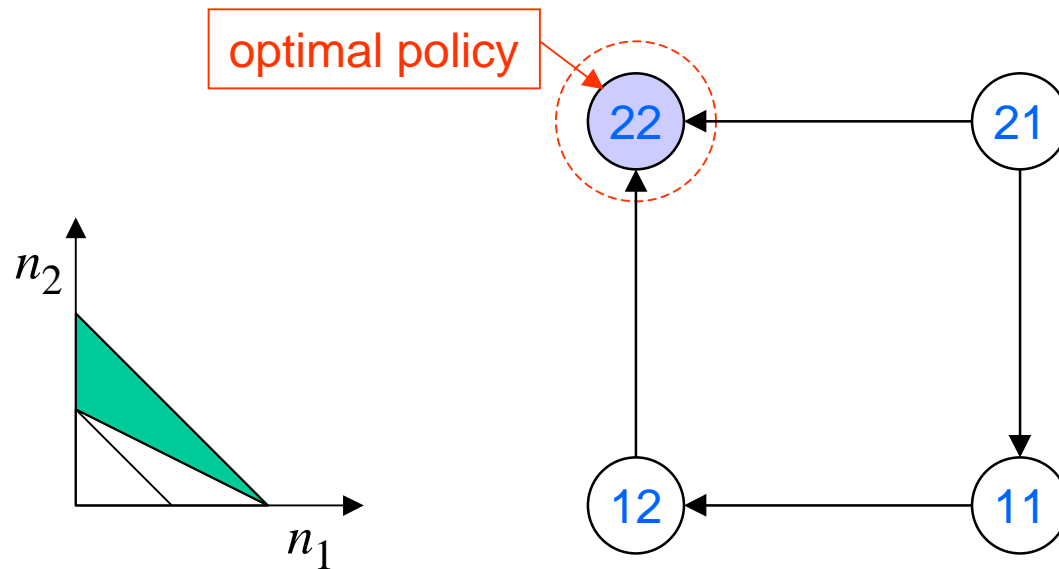
$$(B) \quad n_1 b + n_2 \phi b < 1$$



## Individually optimal actual bit rates (4)

$$(B^c) \quad n_1 b + n_2 \phi b > 1$$

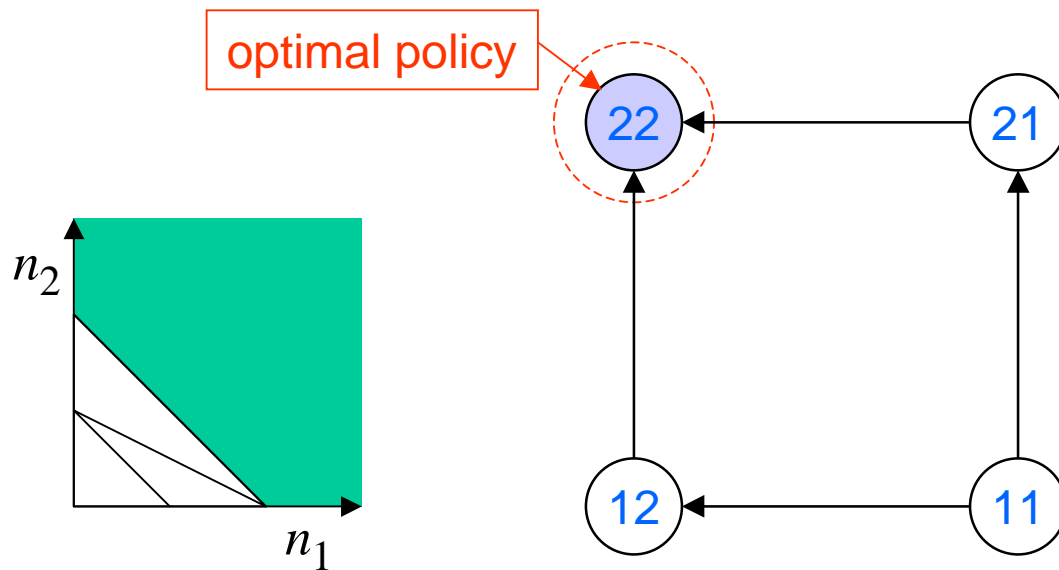
$$(C) \quad n_1 b + n_2 b < 1$$





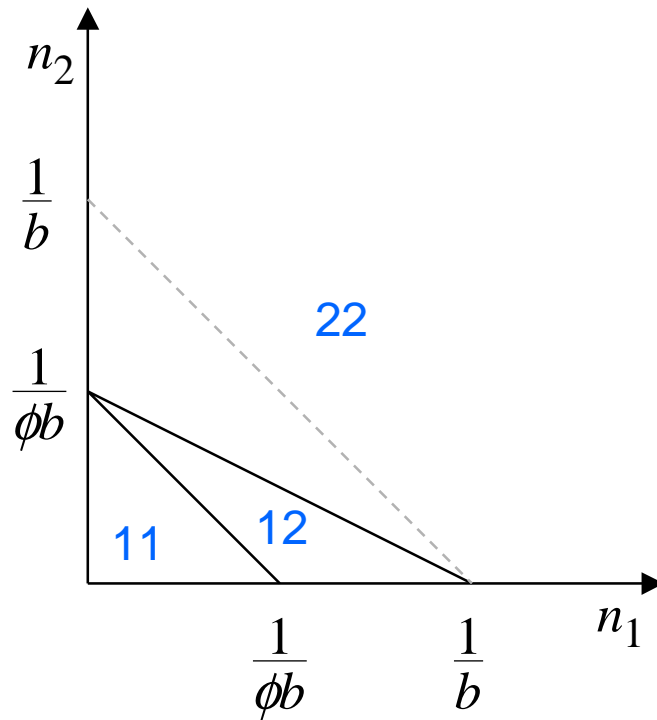
## Individually optimal actual bit rates (5)

$$(C^c) \quad n_1 b + n_2 b > 1$$

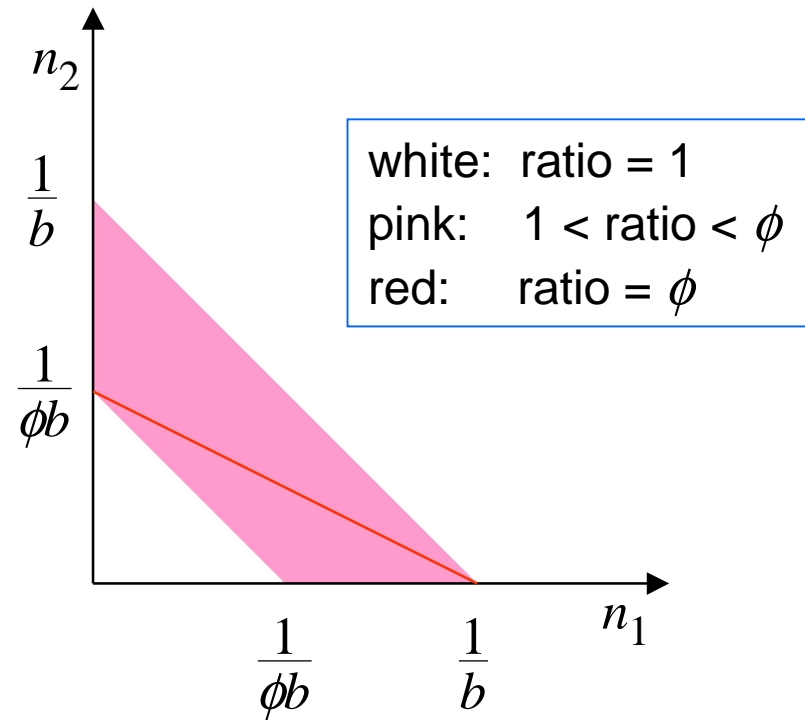


## Individually optimal actual bit rates (6)

Optimal policy



Actual bit rate ratio

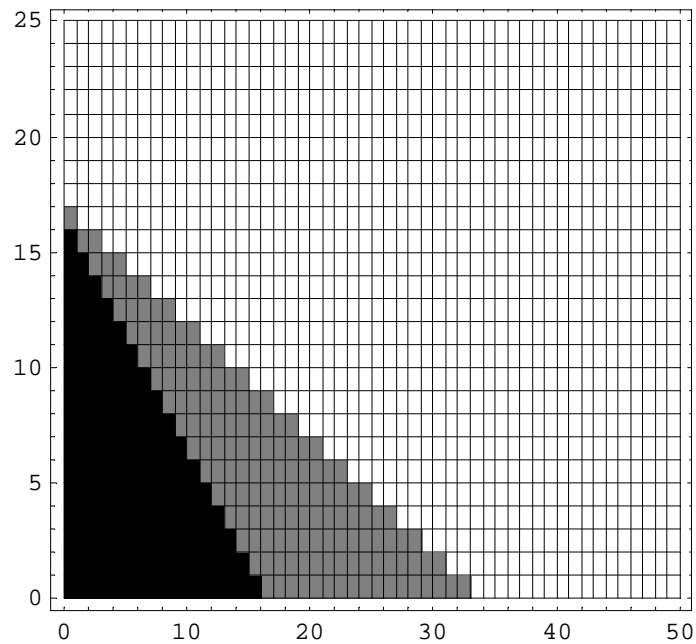


NBR-1 = 0.04  
NBR-2 = 0.08

## Example 1

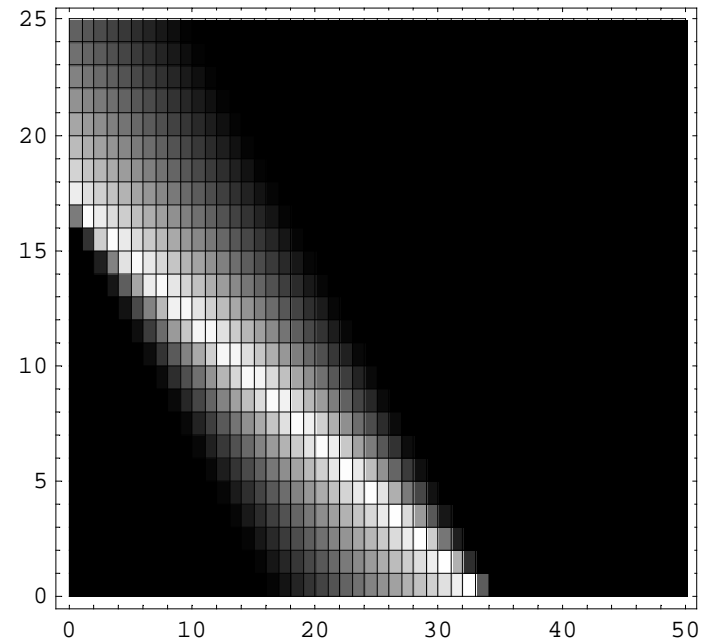
$b = 0.04/\sqrt{2}$   
 $\phi = 2$

Optimal policy



$$\frac{1}{\phi b} = 17.7 \quad \frac{1}{b} = 35.4$$

Actual bit rate ratio

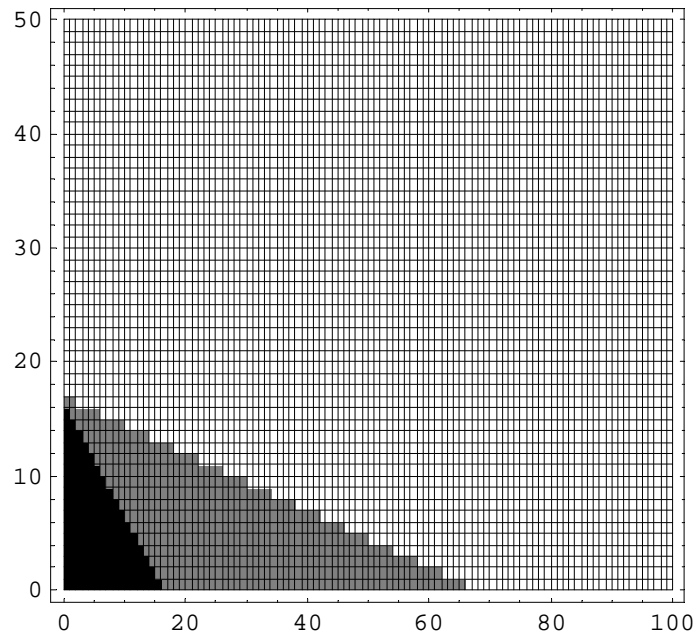


NBR-1 = 0.02  
NBR-2 = 0.08

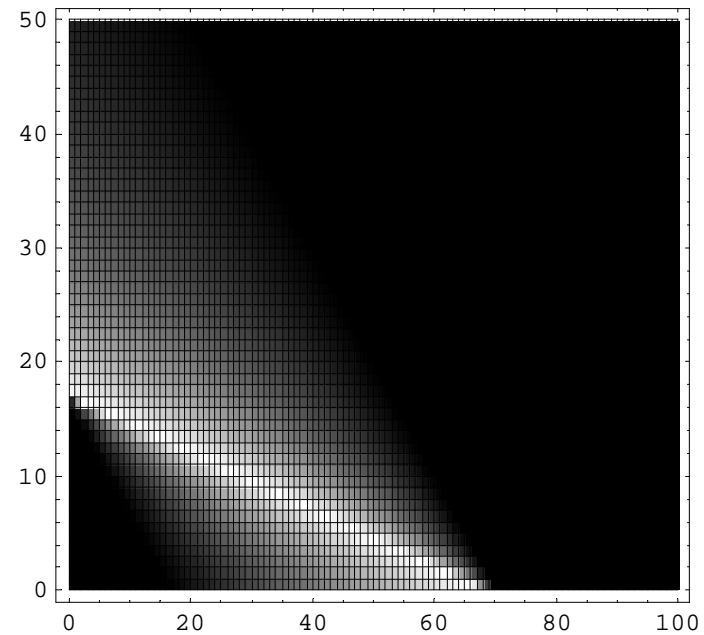
## Example 2

$b = 0.02/\sqrt{2}$   
 $\phi = 4$

Optimal policy



Actual bit rate ratio

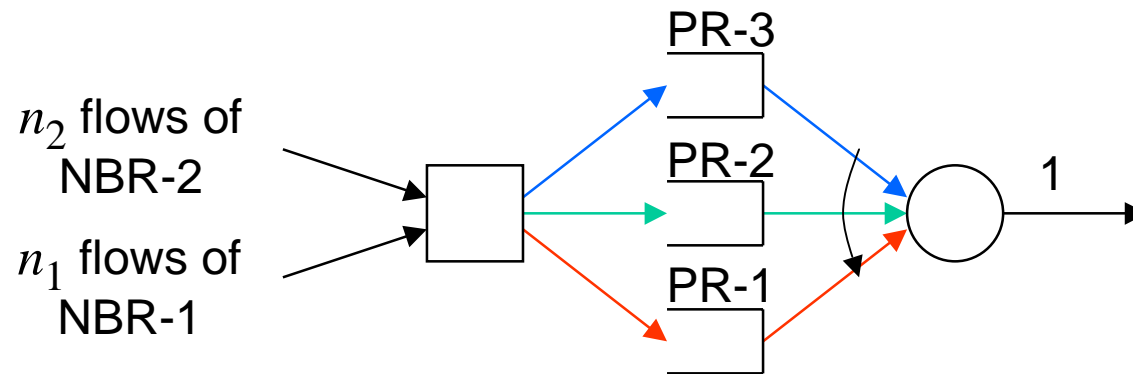


$$\frac{1}{\phi b} = 17.7 \quad \frac{1}{b} = 70.7$$

## Contents

- Two priority classes
- Three priority classes

## Flow model for three priority classes



	NBR-1	NBR-2
PR-3	$ABR < b/2$	$ABR < (\phi b)/2$
	$b/2$	$(\phi b)/2$
PR-2	$b/2 < ABR < b$	$(\phi b)/2 < ABR < \phi b$
	$b$	$\phi b$
PR-1	$ABR > b$	$ABR > \phi b$

## Equations for actual bit rates

$$\beta_{13} = \min\left\{\frac{1}{n_{13}+n_{23}}, \frac{b}{2}\right\}$$

$$\beta_{23} = \min\left\{\max\left\{\frac{1}{n_{13}+n_{23}}, \frac{1-n_{13}\frac{b}{2}}{n_{23}}\right\}, \frac{\phi b}{2}\right\}$$

$$\beta_{12} = \min\left\{\frac{1-n_{13}\beta_{13}-n_{23}\beta_{23}}{n_{12}+n_{22}}, b\right\}$$

$$\beta_{22} = \min\left\{\max\left\{\frac{1-n_{13}\beta_{13}-n_{23}\beta_{23}}{n_{12}+n_{22}}, \frac{1-n_{13}\beta_{13}-n_{23}\beta_{23}-n_{12}b}{n_{22}}\right\}, \phi b\right\}$$

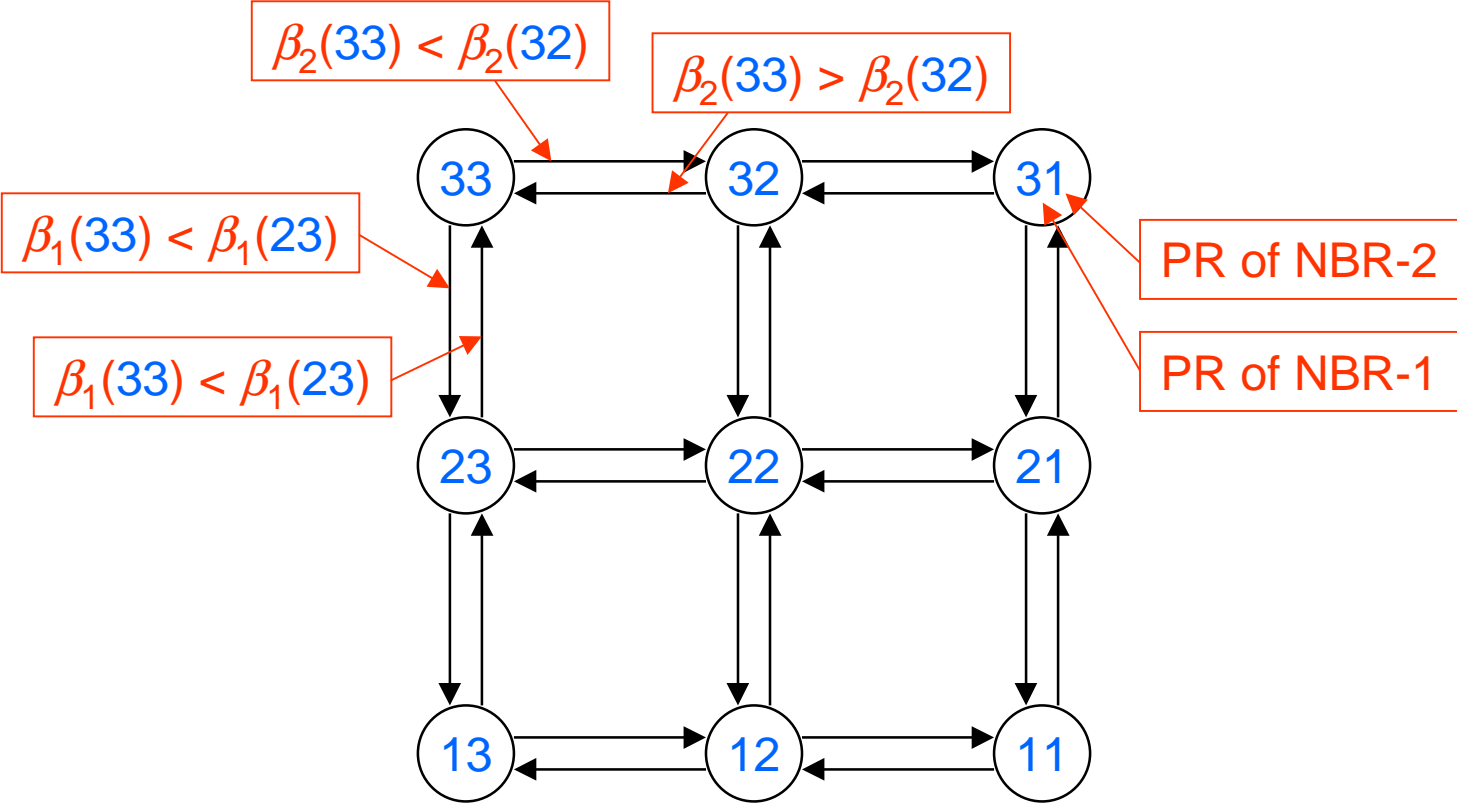
$$\beta_{11} = \max\left\{\frac{1-n_{13}\beta_{13}-n_{23}\beta_{23}-n_{12}\beta_{12}-n_{22}\beta_{22}}{n_{11}+n_{21}}, 0\right\}$$

$$\beta_{21} = \beta_{11}$$

NBR

PR

# Individually optimal actual bit rates



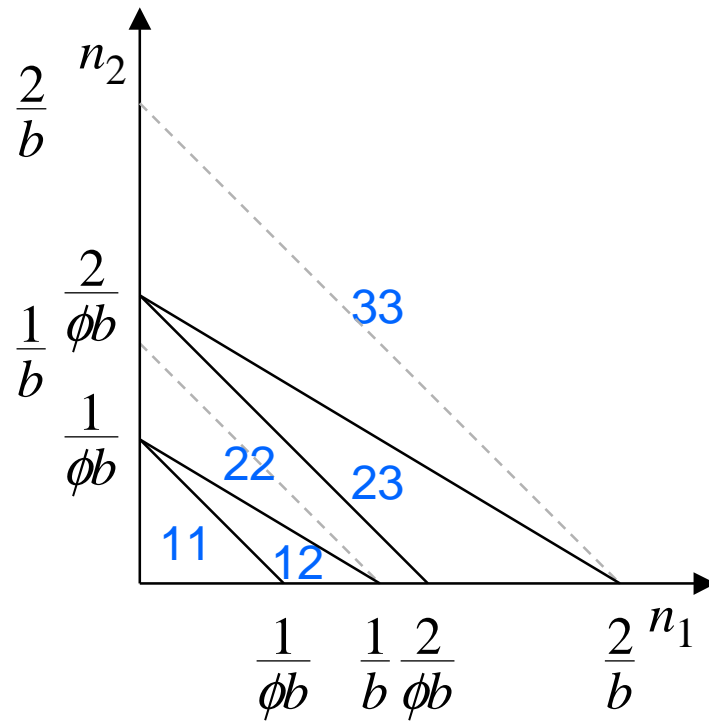
$$\beta_2(33) := \beta_{23}(n_{11}=0, n_{12}=0, n_{13}=n_1, n_{21}=0, n_{22}=0, n_{23}=n_2)$$

$$\beta_2(32) := \beta_{22}(n_{11}=0, n_{12}=n_1, n_{13}=0, n_{21}=0, n_{22}=0, n_{23}=n_2)$$

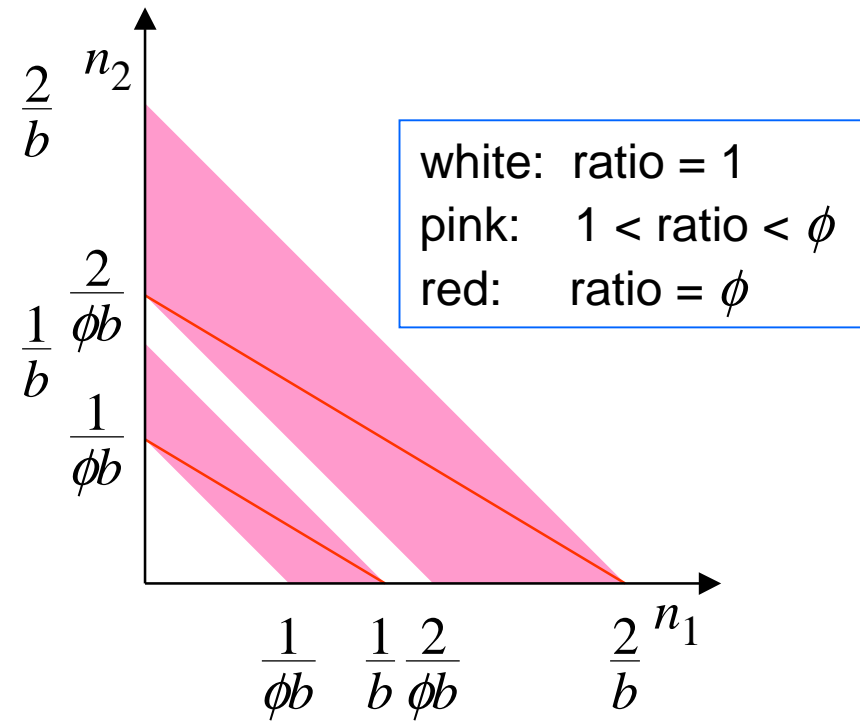


# Case 1: $\phi < 2$

Optimal policy

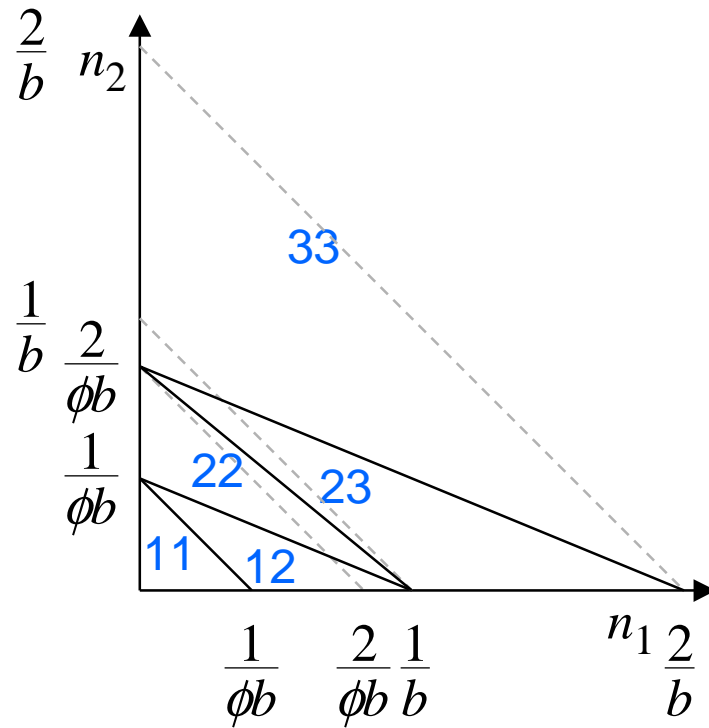


Actual bit rate ratio

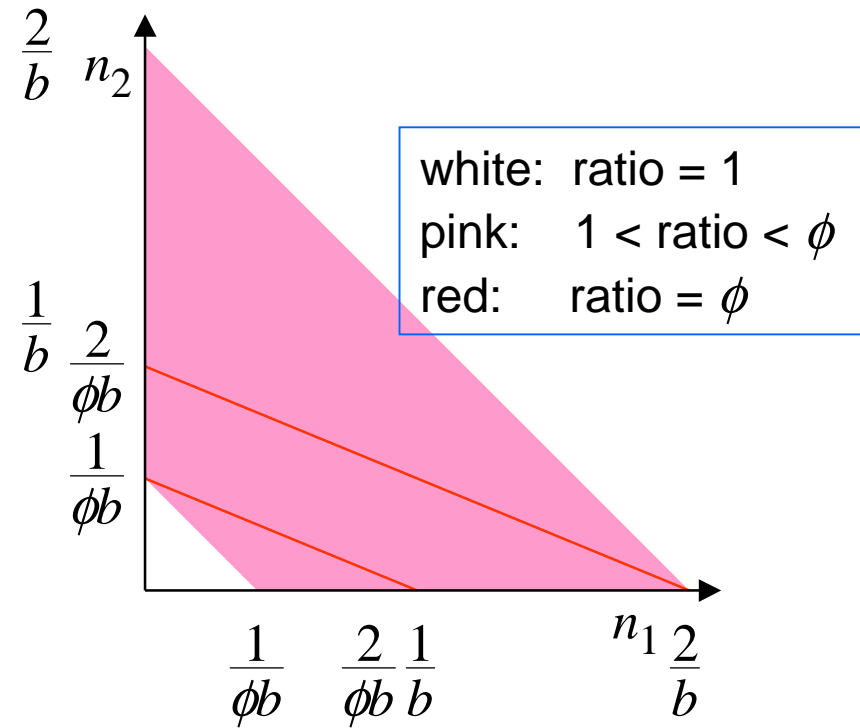


## Case 2: $\phi > 2$

Optimal policy



Actual bit rate ratio



**THE END**

