# Modeling Impact of Delay Spikes on TCP Performance on a Low Bandwidth Link

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# Outline

- RTT spikes & TCP behavior
- renewal model & TCP goodput formula
- simple p-formula
- model validation & evaluation with ns2
- misc. & summary

# TCP in wireless

- TCP designed for fixed networks
- control actions: halving or drastic reduction of sending window
- delayed packets may be taken as lost packets
- timeout timer relies on RTT (round trip time) estimation
- RTTs can be highly variable

# RTT spikes

- sudden increase in RTT = delay spike
- may occur due to:
  - handovers
  - link error recovery
  - scheduling between calls and data (GPRS)
- delay spikes trigger spurious timeouts (packets not lost, simply delayed)

# Illustration of RTT spikes

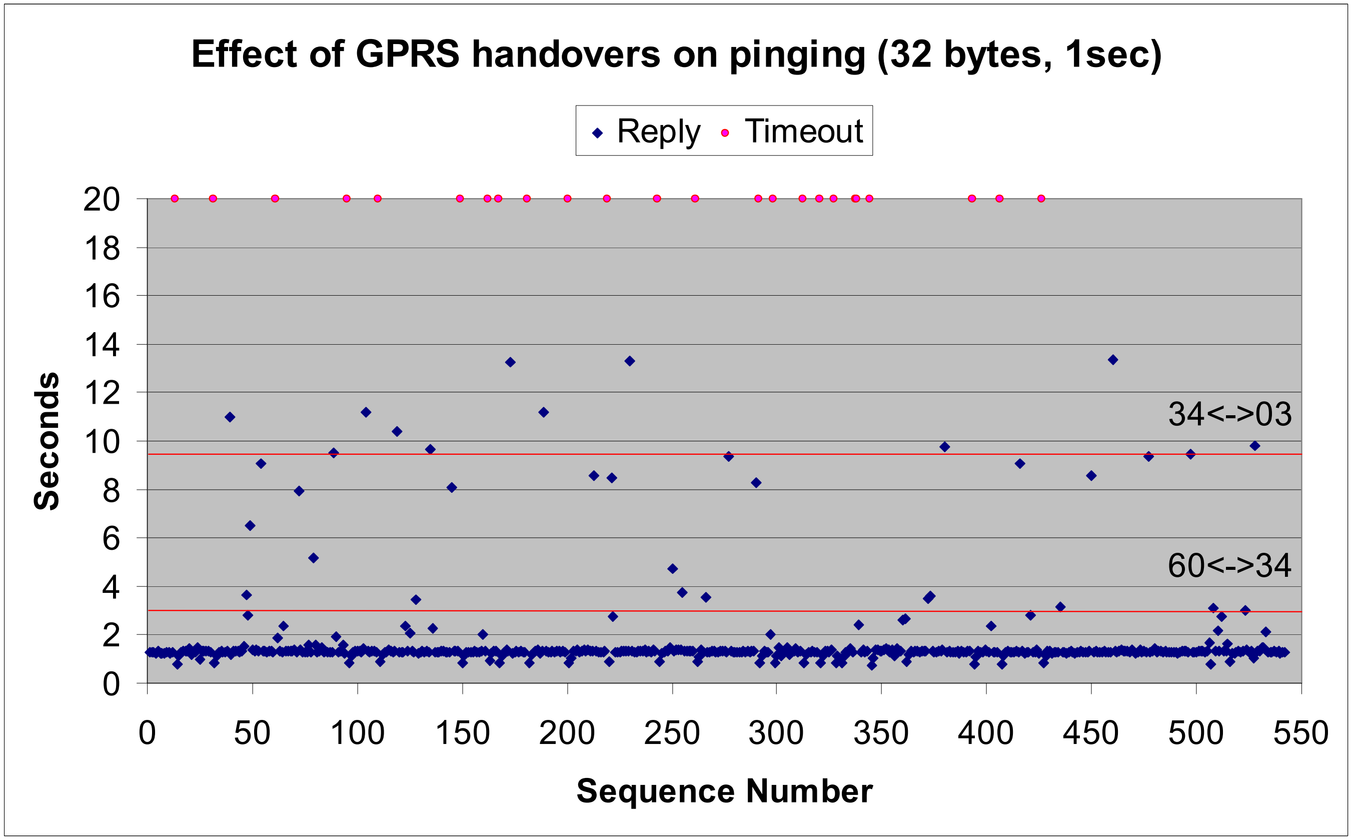


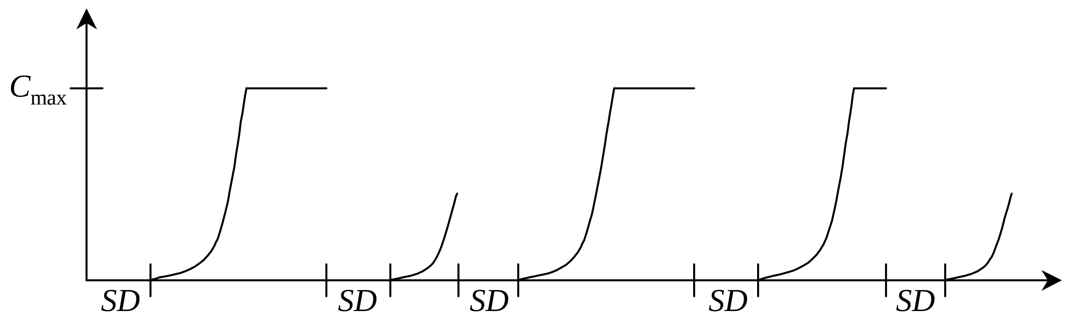

Figure by Andrei Gurtov (see A.Gurtov, Effect of Delays on TCP Performance)

# Our model scenario I

- one TCP source on a low bandwidth wireless link (e.g. GPRS)
- RTT spikes occur
- TCP evolution (grey box approach)
  1. silence for time SD due to an RTT spike
  2. exponential increase in sending window (slow start)
  3. sending rate limited by the low bandwidth
- RTT spikes independent of TCP
- do not model TCP exponential backoff (OK, if SD $< 10$ sec.)
- look at actual sending window (not congestion window, which is larger)
- **OUTCOME: a TCP goodput formula**

# Our model scenario II

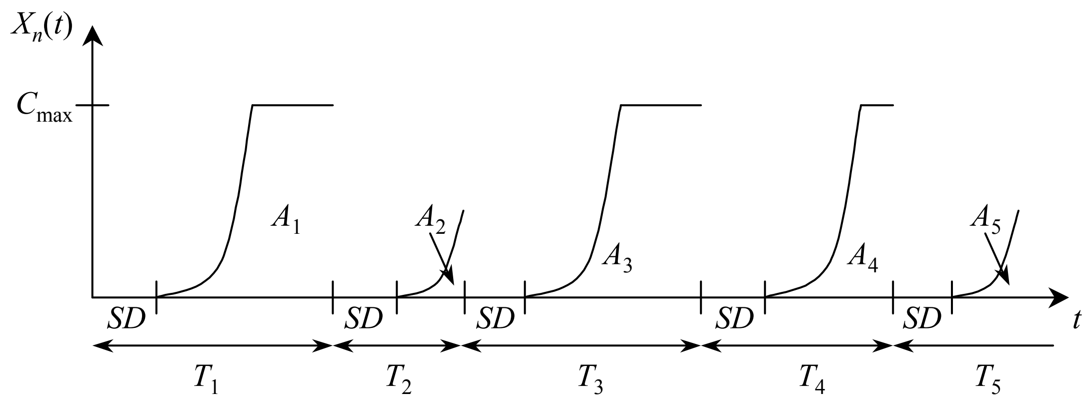
## Illustration of TCP behavior




- silence for time SD due to an RTT spike
- SD= mean value of RTT spike lengths
- exponential increase in sending window (slow start)
- sending rate limited by the low bandwidth

# Renewal reward model

$$\begin{aligned} Z_n &= n\text{th RTT spike, renewal time} \\ T_n &= Z_n - Z_{n-1}\text{, time between spikes} \\ X_n &= \text{TCP sending window} \\ A_n &= \#\text{ of packets sent} = \int_0^{T_n} X_n(u)du\text{, the reward} \end{aligned}$$




$$X_n(t) = \begin{cases} 0, & t < SD, \\ 2^{\frac{t-SD}{\text{RTT}}}, & SD \le t \le L + SD, \\ C_{\max}, & t > L + SD, \end{cases} \quad L = \frac{RTT}{\ln 2} \ln C_{\max}$$

# Goodput with i.i.d. spike intervals

- Assume $T_n$s i.i.d. with probability density $f_T$, $E(T) = \infty$
- $A_t =$ cumulative total reward up to time $t$, Renewal reward theorem:

$$\lim_{t \to \infty} \frac{E(A_t)}{t} = \frac{E(A)}{E(T)}$$

## Goodput formula

$$G_{\mathsf{TCP}} = \frac{E(A)}{E(T)}$$

$$E(A) = \int_0^T X(t)dt$$

$$= \frac{\mathsf{RTT}}{\ln 2}\left(\int_{SD}^{L+SD} (2^{\frac{y-SD}{\mathsf{RTT}}} - 1) f_T(y)dy + (C_{\max} - 1)\int_{L+SD}^{\infty} f_T(y)dy\right) + C_{\max}\int_{L+SD}^{\infty}(y - L - SD) f_T(y)dy,$$

where $L =$ time to reach $C_{\max}$

input: SD, RTT, $C_{\max}$, spike distribution

# Goodput with exponential intervals

Assume: RTT spike intervals from exponential distribution with parameter $1/E(T)$

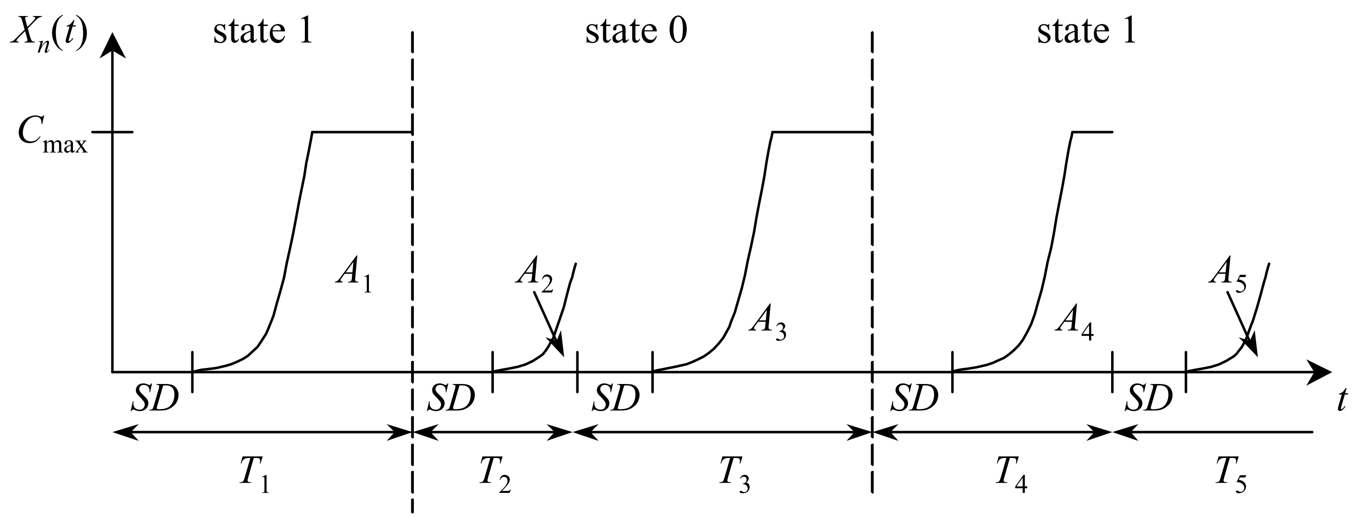
**Exponential goodput formula**

$$G_{\mathsf{TCP}} = \frac{E(A)}{E(T)} = \frac{e^{-SD/E(T)}\left(\frac{\mathsf{RTT}}{E(T)} - \ln 2 C_{\max}^{1-\mathsf{RTT}/(E(T)\ln 2)}\right)}{\mathsf{RTT}/E(T) - \ln 2}.$$

depends on $\frac{\mathsf{SD}}{E(T)}$, $\frac{\mathsf{RTT}}{E(T)}$ rations and $C_{\max}$

# Markov Renewal Model

- introduce correlations between $T_n$s
- "cycles" no longer i.i.d., but modulated by Markov chain $B_n$
- assume: modulating background process slower than RTT spike process
- simple case: Markov chain with states 0 and 1, invariant distribution $\pi$



# Markov Goodput Formula

Markov Renewal Reward Theorem:

$$\lim_{t\to\infty} \frac{1}{t} A_t = \frac{E_\pi(A)}{E_\pi(T).}$$

Backgound $B_n$ with probability density $f_0(t)$ at state 0 and $f_1(t)$ at state 1 gives

$$G_{TCP} = \frac{E_\pi\left(\int_0^T X(u)du\right)}{E_\pi(T)}$$
$$= \frac{\pi(0)\int_0^\infty f_0(t)\int_0^t X(u)du\, dt + \pi(1)\int_0^\infty f_1(t)\int_0^t X(u)du\, dt}{\pi(0)\int_0^\infty f_0(t)tdt + \pi(1)\int_0^\infty f_1(t)tdt},$$

$\pi(0)$ and $\pi(1)$ equilibrium probabilities of states 0 and 1

# Markov with exponential intervals

Take exponential probability densities

$$f_0(t) = 1/\mu_0 e^{-t/\mu_0} \quad \text{and} \quad f_1(t) = 1/\mu_1 e^{-t/\mu_1}$$

to get

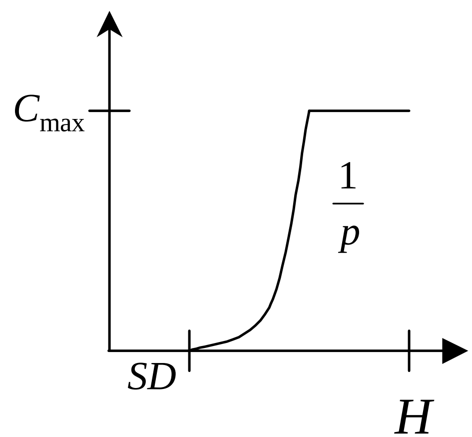
$$G_{TCP} = \frac{\pi_0 e^{-SD/\mu_0} \mu_0 \frac{\mathsf{RTT} - \mu_0 \ln 2 C_{\max}^{1-\frac{\mathsf{RTT}}{\mu_0 \ln 2}}}{\mathsf{RTT} - \mu_0 \ln 2} + \pi_1 e^{-SD/\mu_1} \mu_1 \frac{\mathsf{RTT} - \mu_1 \ln 2 C_{\max}^{1-\frac{\mathsf{RTT}}{\mu_1 \ln 2}}}{\mathsf{RTT} - \mu_1 \ln 2}}{\pi_0 \mu_0 + \pi_1 \mu_1}.$$

# Simple p-formula I

- $P\{\text{packet experiences an RTT spike}\} = p$
- $A_n \sim \text{geom}(p)$, so $E(A_n) = 1/p$
- TCP sending rate at time $t$ (after SD has ellapsed)

$$g(t) = \min(2^{t/\mathsf{RTT}-1/2}, C_{\max})$$

- $H$ = time to sent $1/p$ packets



# Simple p-formula II

$H$ = time to sent $1/p$ packets

**Throughput:**

$$T_{\mathsf{TCP}} = \frac{1/p}{H + \mathsf{SD}}$$

**Result:**

If $p > \frac{\ln 4}{\mathsf{RTT}(2C_{\max} - \sqrt{2})}$

$$T_{\mathsf{TCP}} = \frac{\ln 2}{p(\mathsf{RTT}\ln(1 + \sqrt{2}\ln 2/(p\mathsf{RTT})) + \mathsf{SD}\ln 2}$$

otherwise (TYPICAL CASE!)

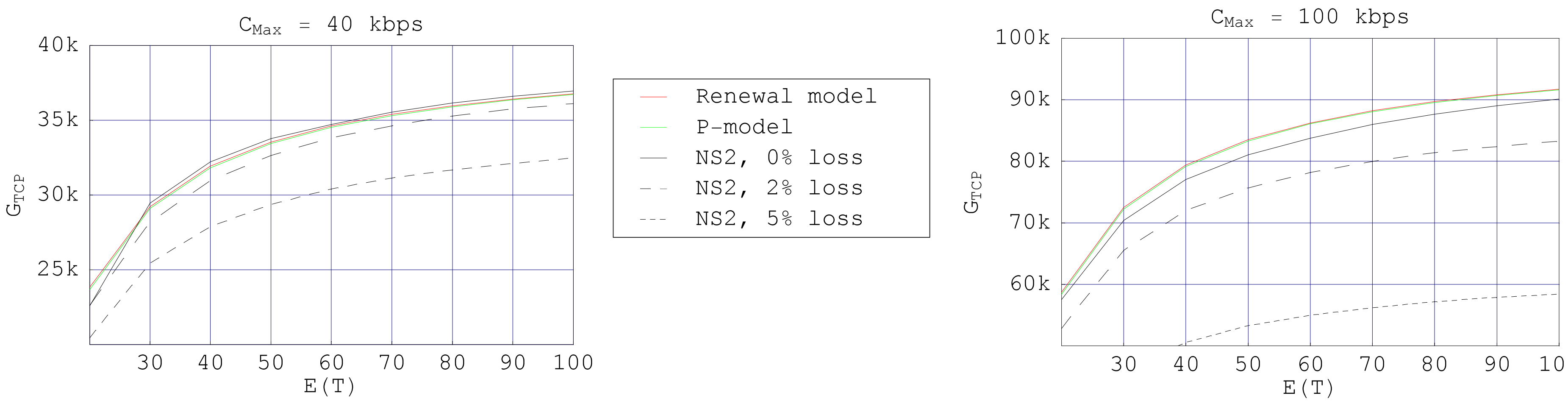
$$T_{\mathsf{TCP}} = \frac{C_{\max}\ln 4}{\ln 4 + p\mathsf{RTT}(\sqrt{2} + C_{\max}(\ln 2 - 2 + 2\ln C_{\max})) + \mathsf{SD}pC_{\max}\ln 4}$$

# Validation with ns2

- RTT spike generator by A. Gurtov
  - turns link on/off for random periods at random intervals
- TCP SACK
- parameters:
  - packet size = 576 bytes
  - link one-way delay = 100 ms
  - RTT = $2\times$ link delay + packet transmission time

# Long spike duration

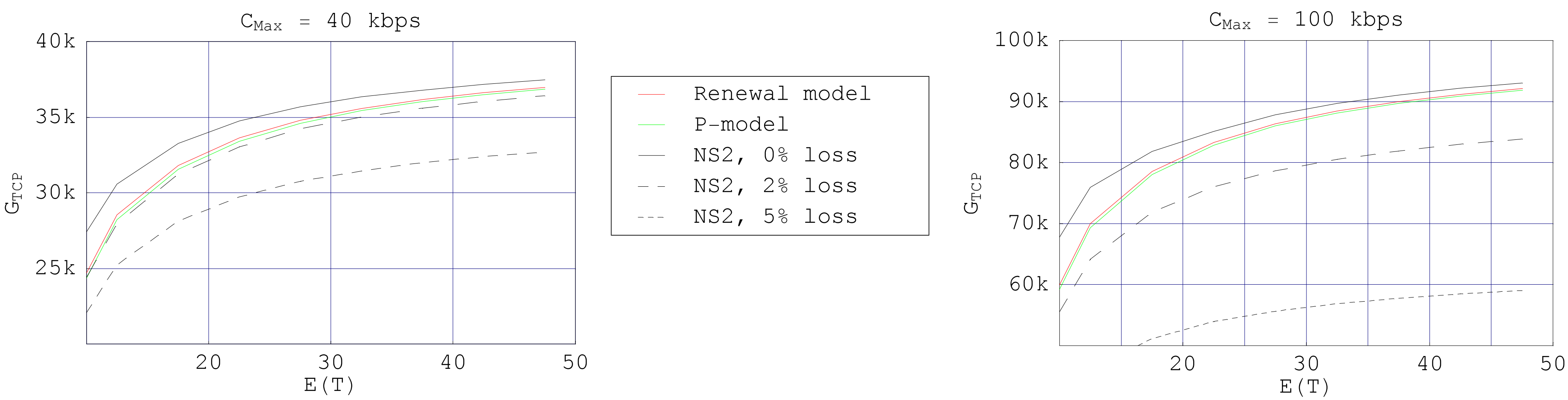
RTT spikes $\sim$ Uni[5,10], SD =7.5 s.




- Low bandwidth:
  - excellent agreement
  - OK, if 2% losses (Note: losses not modelled)
- Higher bandwidth:
  - Nice (although model made for low bandwidth)

# Short spike duration

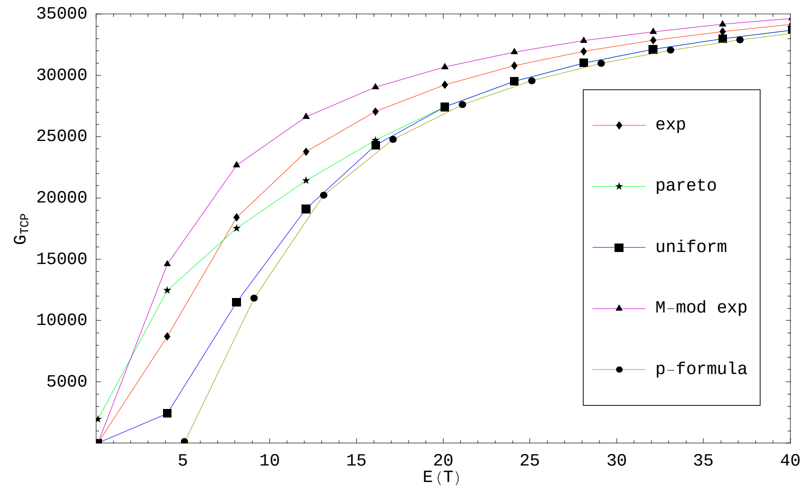
RTT spikes $\sim$ Uni[1,5], SD =3 s.




- TCP learns the spike process, less timeouts $\Rightarrow$ model underestimates
- now losses have a bigger impact

# Distribution sensitivity

- pareto with $\frac{ab^a}{x^{a+1}}$, $a = 1.5$, $x \geq b$
- Markov with exp distributions, $\pi_0 = \pi_1 = 1/2$ and $\mu_0 = 1/4E(T)$, $\mu_1 = 1.75E(T)$
- $C_{\text{max}}$ =8.68 packets, RTT=720 ms and SD=5 sec.




- **bursty spikes give better goodput!**
- **p-formula is a lower bound!**

# Summary

- TCP goodput formula for low bandwidth links with RTT spikes
- grey box approach to TCP
- correlations to RTT spike process using Markov modulation
- model not complicated, allows to ellaborate more on spike process
- simple p-formula (gives lower bounds)
- excellent agreement with ns2
- future
  - include packet losses (congestion control phase)
  - add distribution on RTT spike lengths