Traffic Matrix Estimation

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Background

- Traffic matrix gives the traffic demand between each origin-destination pair in the network
- Knowledge of traffic matrices is important in capacity planning, network management, pricing, traffic engineering.
- However, Traffic matrices are usually not directly available in IP networks
- What is available:
  - Link load measurements $\boldsymbol{y}$ (from SNMP data),
  - routing tables $\boldsymbol{A}$

Techniques to Estimate the Traffic Matrix

- Direct measurement
  - Cisco Netflow
  - MPLS
- Gravitation model
- Linear programming (avg. error: 170%)
- Network tomography (10-25%)
- Bayesian inference (20-45%)

⟶ Average errors from Medina et al. "Traffic matrix estimation: Existing techniques and new directions" 2002.

Gravitation model

- Traffic volume x between an OD-pair ij is proportional to:

  $\rightarrow$ O_i, total traffic originating from node i

  $\rightarrow$ T_j, total traffic terminating at node j

  $\rightarrow$ Some distance function f_{ij}

$$\boldsymbol{x}_{ij} = \frac{O_i T_j}{f_{ij}}$$

- Used to obtain prior distributions as starting points for other algorithms.

Network Tomography

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{y}$$

Where $\boldsymbol{y}$ is the vector of link count measurements, $\boldsymbol{A}$ is routing matrix, and $\boldsymbol{x}$ is the traffic matrix written as a column vector

- Since there are n OD pairs and significantly smaller number m of links, the problem is highly under-constrained for solving the traffic matrix $\boldsymbol{x}$

$\rightarrow$ Many solutions for $\boldsymbol{x}$ yield the measured link counts $\boldsymbol{y}$.

$\rightarrow$ Given a prior distribution some solutions are more probable than others.

Bayesian Inference

- Computes conditional probability distribution for OD-pair traffic demands, given the link counts and prior distribution.

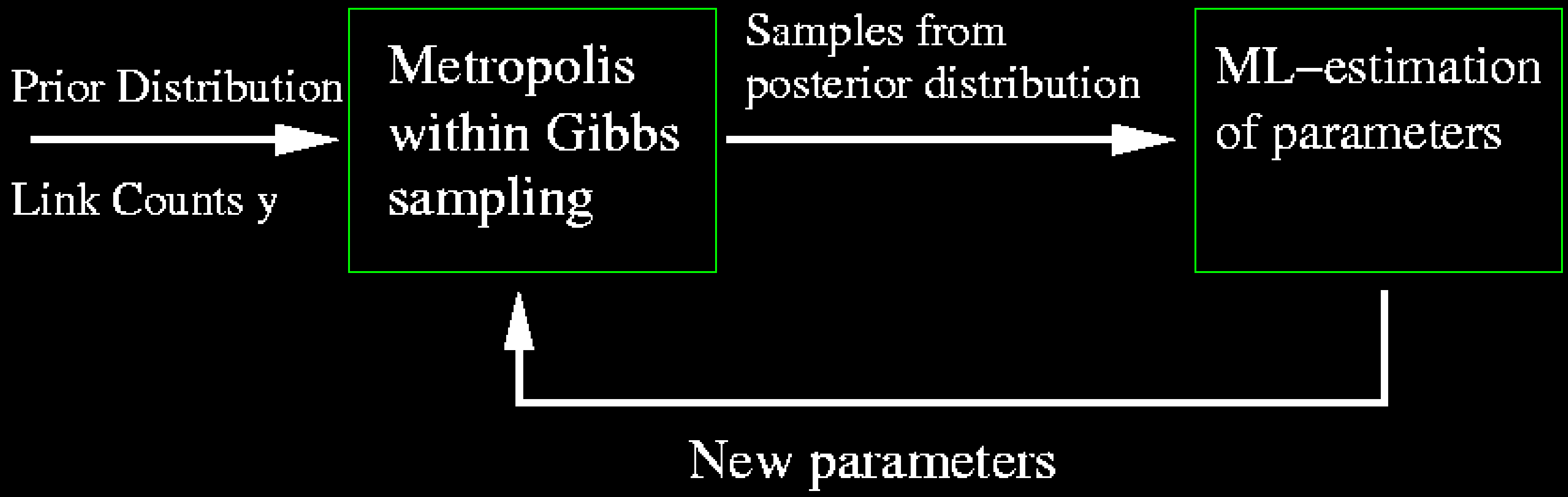
$$p(\boldsymbol{x}, \boldsymbol{\Lambda}) = p(\boldsymbol{\Lambda}) \prod_{a=1}^{n} \frac{\lambda_a}{x_a!} e^{-\lambda_a}$$

- Mean rates $\boldsymbol{\Lambda} = \lambda_1, ..., \lambda_n$ are unknown
- Analytical computations for Posterior distribution are difficult

$\rightarrow$ Markov Chain Monte Carlo simulation for posterior distribution $p(\boldsymbol{x}, \boldsymbol{\Lambda} | \boldsymbol{y})$

Iterative Bayesian estimation

- Vaton, Gravey 2002



Conditional normal distribution

- Traffic matrix $\boldsymbol{x}$ is a multivariate gaussian variable X with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

$$\begin{aligned} \boldsymbol{x} &\sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \\ \boldsymbol{y} &\sim N(\boldsymbol{A}\boldsymbol{\mu}, \boldsymbol{A}\boldsymbol{\Sigma}\boldsymbol{A}^{\mathrm{T}}) \end{aligned}$$

$$f(\boldsymbol{x}) \sim \exp(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})).$$

- We have a prior distribution with estimates $(\boldsymbol{m}, \boldsymbol{C})$ for parameters $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. And let use the notation $\boldsymbol{B} = \boldsymbol{C}^{-1}$
- $\boldsymbol{y}^1, \boldsymbol{y}^2, \ldots, \boldsymbol{y}^D$ are the link load measurements.

- Say we have n OD pairs and m links.

  $\rightarrow$ Routing matrix $\boldsymbol{A}$ has n columns and m rows

  $\rightarrow$ $\boldsymbol{x}$ is an n-vector, $\boldsymbol{y}$ is an m-vector

- Make the partition

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \end{pmatrix} \qquad \boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 \end{pmatrix}$$

  so that $\boldsymbol{A}_1$ is $m \times m$ matrix, $\boldsymbol{A}_2$ is $m \times (n-m)$ and $\boldsymbol{x}_1, \boldsymbol{x}_2$ are m-vector and $(n-m)$-vector respectively.

- Now we can write

$$\begin{aligned} \boldsymbol{A}\boldsymbol{x} &= \boldsymbol{A}_1\boldsymbol{x}_1 + \boldsymbol{A}_2\boldsymbol{x}_2 = \boldsymbol{y} \\ \boldsymbol{x}_1 &= \boldsymbol{A}_1^{-1}(\boldsymbol{y} - \boldsymbol{A}_2\,\boldsymbol{x}_2) \end{aligned}$$

$\rightarrow$ We can substitute this expression for $\boldsymbol{x}_1$ in $f(\boldsymbol{x})$.

- Making the partition and substitution, the exponent of $f(\boldsymbol{x})$ becomes

$$\begin{aligned}&\boldsymbol{x}_2^{\mathrm{T}}(\boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11}\boldsymbol{A}_1^{-1}\boldsymbol{A}_2 - \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{12} - \boldsymbol{B}_{21}\boldsymbol{A}_1^{-1}\boldsymbol{A}_2 + \boldsymbol{B}_{22})\,\boldsymbol{x}_2\\ &\quad+\boldsymbol{x}_2^{\mathrm{T}}((\boldsymbol{B}_{21} - \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11})(\boldsymbol{A}_1^{-1}\boldsymbol{y} - \boldsymbol{m}_1) + (\boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{12} - \boldsymbol{B}_{22})\boldsymbol{m}_2)\\ &\qquad+(\text{transpose})\boldsymbol{x}_2 + \text{constant}.\end{aligned}$$

- This can be written as complete square of the form

$$\begin{aligned}&(\boldsymbol{x}_2 - \tilde{\boldsymbol{m}}_2)^{\mathrm{T}}\tilde{\boldsymbol{C}}_{22}^{-1}(\boldsymbol{x}_2 - \tilde{\boldsymbol{m}}_2) + \text{constant}\\ =\;& \boldsymbol{x}_2^{\mathrm{T}}\tilde{\boldsymbol{C}}_{22}^{-1}\boldsymbol{x}_2 - (\boldsymbol{x}_2^{\mathrm{T}}\tilde{\boldsymbol{C}}_{22}^{-1}\tilde{\boldsymbol{m}}_2 + (\text{transpose})\boldsymbol{x}_2) + \text{constant}\end{aligned}$$

- From which we can pick out terms $\tilde{C}_{22}^{-1}$ and $\tilde{C}_{22}^{-1}\tilde{m}_2$ and solve for $\tilde{\boldsymbol{m}}$

- For each measurement $\boldsymbol{y}^i$ we obtain $\tilde{\boldsymbol{m}}$ using prior estimates $(\boldsymbol{m}, \boldsymbol{C})$ and routing matrix $\boldsymbol{A}$.

$$\tilde{\boldsymbol{m}} = \boldsymbol{G}\boldsymbol{y} + \boldsymbol{H}\boldsymbol{m}$$

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{A}_1^{-1} + \boldsymbol{A}_1^{-1}\boldsymbol{A}_2\tilde{C}_{22}(\boldsymbol{B}_{21} - \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11})\boldsymbol{A}_1^{-1} \\ -\tilde{C}_{22}(\boldsymbol{B}_{21} - \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11})\boldsymbol{A}_1^{-1} \end{pmatrix}$$

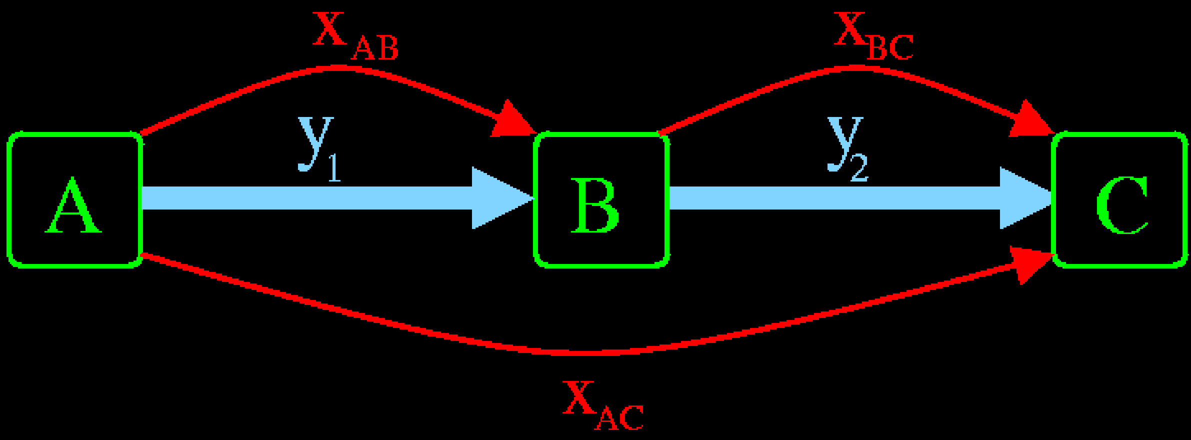
$$\boldsymbol{H} = \begin{pmatrix} -\boldsymbol{A}_1^{-1}\boldsymbol{A}_2\tilde{\boldsymbol{C}}_{22}(\boldsymbol{B}_{21} - \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11}) & \boldsymbol{A}_1^{-1}\boldsymbol{A}_2\tilde{\boldsymbol{C}}_{22}(\boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{12} - \boldsymbol{B}_{22}) \\ \tilde{\boldsymbol{C}}_{22}(\boldsymbol{B}_{21} - \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11}) & -\tilde{\boldsymbol{C}}_{22}(\boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{12} - \boldsymbol{B}_{22}) \end{pmatrix}$$

$$\tilde{\boldsymbol{C}}_{22} = (\boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{11}\boldsymbol{A}_1^{-1}\boldsymbol{A}_2 + \boldsymbol{A}_2^{\mathrm{T}}(\boldsymbol{A}_1^{-1})^{\mathrm{T}}\boldsymbol{B}_{12} + \boldsymbol{B}_{21}\boldsymbol{A}_1^{-1}\boldsymbol{A}_2 + \boldsymbol{B}_{22})^{-1}$$

Where $\tilde{\boldsymbol{C}}_{22}$ is the part of the conditional covariance matrix $\tilde{\boldsymbol{C}}$ that corresponds to $\boldsymbol{x}_2$.

- The new estimate for $\boldsymbol{m}$ is the sample mean of the $\tilde{\boldsymbol{m}}$

Example



$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{A}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \boldsymbol{A}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\boldsymbol{x}_1 = \begin{pmatrix} x_{AB} \\ x_{BC} \end{pmatrix} \qquad \boldsymbol{x}_2 = x_{AC}$$

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \quad \Leftrightarrow \quad \begin{array}{l} y_1 = x_{AB} + x_{AC} \\ y_2 = x_{BC} + x_{AC} \end{array}$$

$$\begin{aligned} \boldsymbol{x}_1 &= \boldsymbol{A}_1^{-1}(\boldsymbol{y} - \boldsymbol{A}_2\,\boldsymbol{x}_2) \\ x_{AB} &= y_1 - x_{AC} \\ x_{BC} &= y_2 - x_{AC} \end{aligned}$$

$$\begin{aligned} \boldsymbol{m}^{i+1} &= \boldsymbol{G}\boldsymbol{y} + \boldsymbol{H}\boldsymbol{m} \\ &= \begin{pmatrix} y_1 - \frac{c_1^{-2}}{c_1^{-2}+c_2^{-2}+c_3^{-2}} y_1 - \frac{c_2^{-2}}{c_1^{-2}+c_2^{-2}+c_3^{-2}} y_2 \\ y_2 - \frac{c_1^{-2}}{c_1^{-2}+c_2^{-2}+c_3^{-2}} y_1 - \frac{c_2^{-2}}{c_1^{-2}+c_2^{-2}+c_3^{-2}} y_2 \\ \frac{c_1^{-2}}{c_1^{-2}+c_2^{-2}+c_3^{-2}} y_1 + \frac{c_2^{-2}}{c_1^{-2}+c_2^{-2}+c_3^{-2}} y_2 \end{pmatrix} \\ &+ \tilde{\boldsymbol{C}}_{22} \begin{pmatrix} c_1^{-2} m_1 + c_2^{-2} m_2 - c_3^{-2} m_3 \\ c_1^{-2} m_1 + c_2^{-2} m_2 - c_3^{-2} m_3 \\ -c_1^{-2} m_1 - c_2^{-2} m_2 + c_3^{-2} m_3 \end{pmatrix} \end{aligned}$$

$$y_1 \sim N(10, 2) \qquad y_2 \sim N(11, 2)$$

For example:

$$\boldsymbol{\mu} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

	prior	1.iteration	2.iteration
$\boldsymbol{m}$	$\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$
$\boldsymbol{C}$	$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- How good is this solution?

Thank You for your attention.