Impact of Multidirectional Forwarding on the Capacity of Large Wireless Networks

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***Abstract*—We consider the capacity problem in large wireless multihop networks by separating the problem into macroscopic and microscopic level subproblems. At the macroscopic level, the task is to determine the routes that are geometric curves. At the microscopic level, we need to forward the traffic according to its directional distribution that results from the macroscopic level routes. Previous studies have treated the macroscopic level problem simply as that of load balancing, implying the use of time sharing to serve the traffic flowing in different directions. We study how a scheduling that truly interleaves the traffic flows in different directions affects the macroscopic level problem. We restrict the macroscopic routes to certain simple path sets. This together with earlier results on the microscopic level forwarding capacity allows us to obtain new results on the macroscopic level capacity that demonstrate the gains from multidirectional forwarding.**

***Index Terms*—Wireless multihop networks, load balancing, forwarding capacity, massively dense networks.**

I. Introduction

THE early papers studying the capacity of wireless multihop networks often attempted to answer the question: "What is the optimal mean number of neighbors for maximizing the one-hop progress of a packet in its desired direction?" The results were often referred to as the "magic numbers", see e.g. [1]. These were later contradicted by studies pointing out that a random geometric network with a fixed mean number of neighbors does not stay connected when the number of nodes in the network, N, increases [2]. Instead, the expected node degree should grow faster than the logarithm of the number of nodes to keep the network asymptotically connected [3]. Famously, in [4] it was shown that in this case randomly located nodes with a common (increasing) transmission range can achieve the throughput of $\Theta(1/\sqrt{N \log N})$ bits per second per node for a randomly chosen destination. However, by using a larger transmission range for access and delivery phases and a shorter (fixed) one for transport phase, a better scaling, $\Theta(1/\sqrt{N})$, can also be achieved [5]. This again opens the optimal neighborhood size for discussion. Additionally, the scaling laws leave the constant factor, i.e., the question of the actual capacity, open. We continue by studying this problem in large networks.

As the size of the network increases, a natural separation of spatial scales emerges, and the network capacity problem can be separated into two independent problems [6], [7]. At the macroscopic level, the underlying network is considered a continuous medium, where the routes are smooth geometric curves [6], [7], [8], [9], [10], [11], and the problem is to define the most efficient routing given the traffic matrix and the constraints set by the microscopic level. At the microscopic level, representing the network from a single node's point of view, the network appears to be infinite [12], and the task is to forward traffic as efficiently as possible according to the macroscopic level routing. Jointly optimizing the macroscopic and microscopic levels allows us to solve the total capacity of the network.

A key assumption made in the macroscopic level studies so far, reducing the macroscopic level problem to simple load balancing, is that the constraints set by the microscopic level do not depend on the directional distribution of the traffic that results from the macroscopic level routing. This could be achieved by determining the maximum density of packet flow in a given direction, and sharing this single-directional forwarding capacity in time between the different directions. The assumption only guarantees a lower bound [12]. The sustainable local traffic load is bounded by a limit, which is a functional of the directional distribution of the traffic, and which is called the multidirectional forwarding capacity (MFC). Already the simple load balancing problem is difficult, and it has not been fully solved, see [6] for close results. Taking into account the multidirectionality complicates the problem considerably.

We study how the previously found new formulation [12] for the microscopic level constraint affects the macroscopic level problem. The goal is to find out the order of magnitude of how much taking into account the multidirectionality may increase the total capacity of the network. Our results demonstrate that a significant gain can indeed be obtained if the traffic flowing in different directions can be interleaved in the microscopic level scheduling. We review the separation of scales in Section II. In Section III, we illustrate how the microscopic level results can be combined with macroscopic level routing to find the network capacity. Section IV concludes the paper.

II. Separation of Scales

We study the networks at the limit where the number of nodes in the network tends to infinity. We look at the network from two different perspectives (see Fig. 1): the perspective of the whole massively dense network, from which a single node is meaningless, and the local perspective, from which the events elsewhere in the infinite network bear no significance. These two viewpoints represent the separation of scales [6]:

1) Macroscopic level routing tries to carry as much traffic

Manuscript received September 25, 2013. The associate editor coordinating the review of this letter and approving it for publication was N. Pappas.

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Digital Object Identifier 10.1109/LCOMM.2013.112613.132168

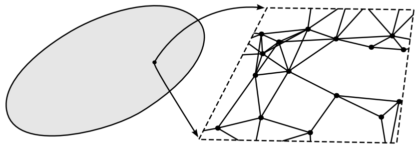


Fig. 1. Macroscopic (left) and microscopic (right) levels.

as possible through the network without exceeding the microscopic level capacity constraints.

2) Microscopic level forwarding aims at coordinating the transmissions so that the packets are relayed hop-by-hop as efficiently as possible and spatial reuse is maximized.

Hence, the forwarding problem at the microscopic level sets an upper bound for the amount of traffic that the routing problem at the macroscopic level is allowed to direct to an area of the network. That the separation of scales is asymptotically exact is intuitively obvious but has not been proven formally.

A. Macroscopic level problem

The interest at the macroscopic level is in the end-to-end paths that are smooth continuous curves. The traffic demand is defined by $\lambda(\boldsymbol{x}_1, \boldsymbol{x}_2)$, where $\lambda(\boldsymbol{x}_1, \boldsymbol{x}_2)\mathrm{d}\boldsymbol{x}_1\mathrm{d}\boldsymbol{x}_2$ is the rate of flow of packets from a differential area element at $\boldsymbol{x}_1$ to another at $\boldsymbol{x}_2$. We call λ the traffic demand density [pkts/s/m^4]. The total offered traffic Λ [pkts/s] in the network domain $\mathcal{A}$ is given by $\Lambda = \iint_{\mathcal{A}} \lambda(\boldsymbol{x}_1, \boldsymbol{x}_2)\mathrm{d}\boldsymbol{x}_1\mathrm{d}\boldsymbol{x}_2$. The traffic demand is satisfied by carrying the packets along the paths of a routing system $\mathcal{P}$. This set of paths contains at least one path for every pair of a source and a destination. The local traffic load in the network that is formed as a result of the routing system $\mathcal{P}$ and traffic demand λ is characterized by the angular flux of packets at point $\boldsymbol{x}$ in direction θ. It is denoted by $\varphi(\boldsymbol{x}, \theta)$ and is equal to the rate [pkts/s/m/rad] at which packets flow in the angle interval $(\theta, \theta+\mathrm{d}\theta)$ across a small line segment of length $\mathrm{d}s$ perpendicular to direction θ divided by $\mathrm{d}s \cdot \mathrm{d}\theta$ in the limit when $\mathrm{d}s \to 0$ and $\mathrm{d}\theta \to 0$. We write $\varphi(\boldsymbol{x}, \theta) = \Phi(\boldsymbol{x}) \cdot f(\theta, \boldsymbol{x})$, where Φ is the scalar flux $\Phi(\boldsymbol{x}) = \int_0^{2\pi} \varphi(\boldsymbol{x}, \theta)\mathrm{d}\theta$, giving the total offered traffic intensity [pkts/s/m], and $f(\theta) = f(\theta, \boldsymbol{x})$ is the directional distribution $\int_0^{2\pi} f(\theta)\mathrm{d}\theta = 1$.

On the macroscopic scale the problem is the following: given a network area $\mathcal{A}$ and the traffic demand density λ, find a routing system $\mathcal{P}$ such that at every point $\boldsymbol{x}$, the local microscopic scale capacity constraint is satisfied. With I^* denoting the microscopic level variable, the MFC, the constraint, as given in [12], reads,

$$\Phi(\boldsymbol{x}; \mathcal{P}) \leq I^*[f(\theta, \boldsymbol{x}; \mathcal{P})] \qquad \forall \boldsymbol{x} \in \mathcal{A}, \tag{1}$$

where the scalar flux Φ and the directional distribution $f(\theta)$ are functions of $\boldsymbol{x}$ as determined by $\mathcal{P}$ (and λ). In particular, to maximize the capacity of the network, we need to find $\mathcal{P}$ such that the above condition is satisfied with the maximal possible scalar multiplier of a given λ. To be explicit, this leads to the following modified load balancing problem:

$$c = \max_{\mathcal{P}} \min_{\boldsymbol{x} \in \mathcal{A}} \; I^*[f(\theta, \boldsymbol{x}; \mathcal{P})]/\Phi(\boldsymbol{x}; \mathcal{P}). \tag{2}$$

If the above maxmin problem is solved with a traffic demand, e.g., with the total offered traffic of $\Lambda_0 = 1$ packet per second, then $\Lambda = c\Lambda_0$ gives the network capacity.

B. Microscopic level problem

From the local perspective, the network appears to be infinite, and only the direction of each packet is relevant. We assume that node locations are distributed according to a homogeneous Poisson point process. In this "locally infinite" network, the traffic is solely relay traffic whose directional distribution appears the same everywhere in the network. Finding a coordinated forwarding scheme that handles traffic with the directional distribution determined by the routing system $\mathcal{P}$ as efficiently as possible is referred to as the microscopic level forwarding problem. The capability of the network to forward traffic with given directional distribution in an infinite network is called the MFC. This microscopic level characteristic sets an upper bound for the allowed macroscopic level load.

The *multidirectional forwarding capacity (MFC)*, I^*, is defined as the maximum sustainable mean density of progress [pkts/m/s], i.e., the density of packets multiplied by their mean velocity in their respective directions.[1] It depends on the directional distribution, $f(\theta)$, of the traffic, and we denote it by $I^*[f(\theta)]$. In general, this functional dependence is unknown but some results, reviewed below, were presented in [12].

The MFC, when studied as a function of the mean node degree, remains zero until the network percolates. According to [13], this happens approximately at 4.51 for nodes having a common transmission range. Additionally, one can state that for any $f(\theta)$

$$I_1^* \leq I^*[f(\theta)] \leq I_\infty^*, \tag{3}$$

where I_∞^* is the limit for uniform directional distribution, $f(\theta) = 1/2\pi$, and I_1^* is the limit with traffic in a single direction with $f(\theta) = \delta(\theta)$, i.e., the Dirac delta function. Note that both depend on the interference model.

Assume that the directional distribution $f(\theta)$ can be expressed as a convex combination

$$f(\theta) = \sum_i a_i g_i(\theta) + b\, h(\theta), \tag{4}$$

where the a_i and b are constants, $\{g_1(\theta), g_2(\theta), \ldots\}$ is a set of directional distributions for which the MFCs are known (denoted by $\{J_1^*, J_2^*, \ldots\}$, respectively), and the remainder term $h(\theta)$ is also a distribution. Each of the components of the sum in (4) can be handled in $\Phi \cdot a_i/J_i^*$ fraction of time. The remainder requires a fraction smaller than or equal to $\Phi \cdot b/I_1^*$. The total traffic can be sustained using time sharing between the components if the sum of the time shares is at most one. Thus, we have the following lower bound for the MFC

$$I^*[f(\theta)] \geq \Big(\sum_i a_i/J_i^* + b/I_1^*\Big)^{-1}.$$

We use the microscopic level results of [12], and the directional distributions for which the MFC is known are the single-, bi- and four-directional balanced traffic patterns (denoted by subindices 1, 2 and 4 respectively). These can be utilized to forward non-balanced four-directional traffic by first separating the four-directional balanced traffic pattern, in which case the remaining traffic equals zero in at least

[1] Information could also be measured, e.g., in bits.

one direction. In the other orthogonal direction, the balanced bidirectional traffic can again be extracted. This only leaves two single-directional orthogonal flows that can be handled using time sharing with single-directional forwarding. This yields the lower bound. By rotating this pattern over all angles $(0, \pi/2)$, a lower bound is obtained for $I^*[f(\theta)]$ for any directional distribution. Explicitly, we have

$$I^*[f(\theta)] \geq \left(\frac{K_1 - K_2}{I_1^*} + \frac{K_2 - K_4}{I_2^*} + \frac{K_4}{I_4^*} \right)^{-1}, \quad (5)$$

where

$$K_i = i \int_0^{2\pi/i} \min_{j=0,\dots,i-1} \{ f(\theta + 2\pi j/i) \} \, \mathrm{d}\theta.$$

The number of parameters needed to describe the microscopic level problem can be reduced by dimensional analysis. The MFC can be expressed as any combination of the parameters having the dimension 1/m/s multiplied by a function of all the independent dimensionless parameters that can be formed. Using the network model of [12], we have

$$I^*[f(\theta)](C, n, \rho) = C\sqrt{n}\, u(\nu; f), \quad (6)$$

where C is the nominal capacity of a link [1/s], n is the density of nodes [1/m^2], and ρ is the transmission range [m]. The dimensionless parameter, $\nu = \pi n \rho^2$, can be interpreted as the mean degree of a node, and u is an unknown dimensionless function.[2]

III. Network Capacity Problem

We have now defined what is needed for studying the total capacity of a network. As a numerical example, we consider a circular disk with area $A = \pi R^2$ and uniform traffic demand density:

$$\mathcal{A} = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \mid |\boldsymbol{x}| < R \right\}, \quad \lambda(\boldsymbol{x}_1, \boldsymbol{x}_2) = \Lambda / A^2.$$

We do not attempt to solve the macroscopic level problem (2) fully but limit ourselves to optimization over some predetermined path sets [7] (see the reference for further details). At the mircoscopic level, we utilize the known results [12] for $u(\nu)$ of (6) that are illustrated in Fig. 3a. The results are based on a slotted-time model where the packets are forwarded using greedy maximum weight scheduling in a network where the nodes are located according to a homogeneous Poisson point process, and a transmission is successful if the receiver is inside the transmission radius of exactly one active transmitter (Boolean interference model).

A. Shortest paths

Shortest paths are commonly used in network studies and provide a baseline for our analysis. As the situation is rotationally symmetric, it is enough to study the positive x-axis. The angular flux on distance r from the origin in direction θ is

$$\varphi_{\mathrm{SP}}(r, \theta) = \frac{\Lambda R (R^2 - r^2)}{A^2} \cdot \sqrt{1 - (r/R)^2 \sin^2 \theta}.$$

[2]It represents the probability that a random node transmits multiplied by the progress of the transmission in units of $1/\sqrt{n}$.

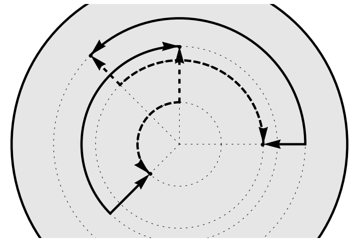


Fig. 2. Inner (dashed) and outer (solid) radial-ring paths between two pairs of nodes.

For the scalar flux to be feasible, it is required that $\Phi(\boldsymbol{x}) \leq I^*$, for all $\boldsymbol{x} \in \mathcal{A}$, where the forwarding capacity is for the used forwarding method.

As the traffic is uniform, and the shortest path routing is bidirectional, i.e., the path from $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$ is the same as the path from $\boldsymbol{x}_2$ to $\boldsymbol{x}_1$, all the traffic can be forwarded using either single- or bidirectional forwarding. The scalar flux has its maximum at the origin, $\Phi_{\mathrm{SP}}(\boldsymbol{0}) = (2/\sqrt{\pi A}) \cdot \Lambda$, that is the bottleneck for both single- and bidirectional forwarding. Hence, from (1) and (6), we have

$$\Lambda \leq C/2 \cdot \sqrt{\pi n A}\, u(\nu)$$

for the total offered traffic.

To make things more concrete, let us further assume that we are studying a network area of 100 km^2 with 0.1 nodes per square meter (10^7 nodes total), the nominal link capacity is equal to one packet per second, and the transmission range is 5 meters. This results in an average of little under eight neighbors per node ($\nu \approx 7.85$ in Fig. 3a). Now, we can transmit at most $\Lambda \approx 230$ packets per second with single-directional forwarding and $\Lambda \approx 330$ packets per second with bidirectional forwarding.

When also four-directional forwarding is possible, the local bottleneck capacity for SP as a function of the distance from the origin (in units of R) can be seen in Fig. 3b. The MFC has been calculated using the lower bound of (5). As there is only little traffic that cannot be forwarded using four-directional forwarding, mostly at the border of the network, the origin remains the bottleneck, and $\Lambda \approx 440$ packets per second. Since the used paths were fixed and the situation is rotationally symmetric, (2) reduces to minimization over r.

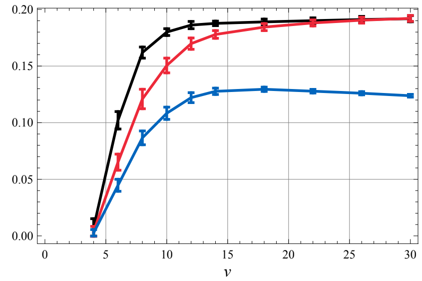
B. Radial-ring paths

In addition to the shortest paths, we study so-called inner and outer radial-ring paths. These path sets, illustrated in Fig. 2, consist of one radial component and one ring component. With the inner radial-ring paths (IRR), the order is chosen so that the ring component closer to the origin is used, and for outer radial-ring paths (ORR), it is the opposite. The idea is to use these paths mixed with the shortest paths to push some of the traffic away from the center of the network.

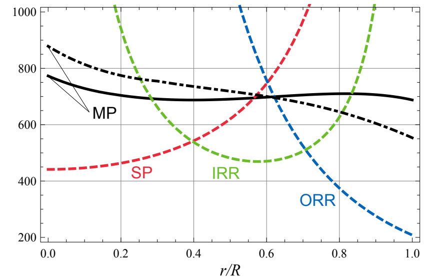
The angular fluxes at $(r, 0)$ in direction θ with radial-ring paths are

$$\varphi_{\mathrm{IRR}}(r, \theta) = \frac{\Lambda \left(\frac{r}{R} - (\frac{r}{R})^3 \right)}{2\pi R} \Big(\mathbb{1}_{\{\theta \in \{0, \pi\}\}} + \pi \cdot \mathbb{1}_{\{\theta \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}\}} \Big),$$

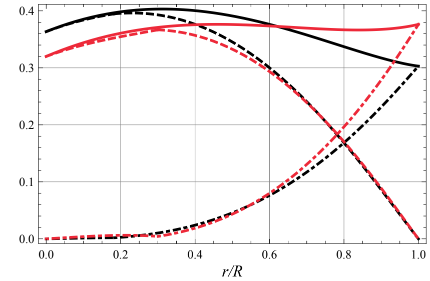
$$\varphi_{\mathrm{ORR}}(r, \theta) = \frac{\Lambda (\frac{r}{R})^3}{2\pi R} \Big(((\tfrac{R}{r})^2 - 1) \cdot \mathbb{1}_{\{\theta \in \{0, \pi\}\}} + \pi \cdot \mathbb{1}_{\{\theta \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}\}} \Big).$$



(a) Dimensionless forwarding capacity, u, as a function of the mean node degree for balanced four-directional traffic (highest), balanced bidirectional traffic (middle), and single-directional traffic (lowest) [12].



(b) Local bottleneck capacity as a function of the distance from the origin for optimal MP (solid), MP with simple load balancing (dot-dashed), and the different elementary path sets (dashed lines).



(c) Traffic load, Φ/Λ, for optimal MP (solid black) and its four- (dashed) and bidirectional (dot-dashed) components. MP with simple load balancing is given for comparison.

Fig. 3. Numerical results.

After the addition of the radial-ring paths, the system is still rotationally symmetric, and it is enough to minimize over r in (2). The maximization over the routing system now involves selecting the optimal convex combination of the path selection probabilities, $\boldsymbol{p}$. Hence the path selection is now randomized, and each packet selects the used path with the given probabilities. The optimal path selection probabilities for multipath (MP) routes are approximately 0.571 (SP), 0.126 (IRR), and 0.303 (ORR). Using this routing system, we can transmit at most $\Lambda \approx 690$ packets per second, cf. Fig. 3b.

If we ignored the different microscopic level performance for different directional distributions and assumed that the MFC is constant, $I^*[f(\theta)] = I$, solving (2) would be equivalent to minimizing the maximum local load, $\min_{\boldsymbol{p}} \max_r \Phi(r)/I$. That is, we would have a simple load balancing problem. Since $K_1 = 1$ in (5), all the traffic can be handled by time sharing between single-directional flows, and I_1^* is a lower bound for $I^*[f(\theta)]$ as given in (3). Assuming that we utilized this time sharing to reduce (2) to pure load balancing, the capacity would be $\Lambda \approx 400$ packets per second. If we still used multidirectional forwarding at the microscopic level whenever possible, the capacity would be $\Lambda \approx 550$ packets per second even though the macroscopic level performance is not optimal, cf. Fig. 3b. The traffic load (Φ/Λ) as a function of the distance from the origin (in units of R) for optimal MP and MP that balances the load [7], and how they separate into four- and bidirectional components, has been compared in Fig. 3c.

IV. Discussion

Our results illustrate the benefit of taking into account the microscopic level dependence on the directional distribution of the traffic when balancing the load on the macroscopic level of a very large wireless network. Significantly higher capacities are achieved by exploiting the fact that traffic flowing in different directions can be interleaved in the microscopic level scheduling. The gain depends on the network model. If the node degree of the transport network had been increased, the difference between bi- and four-directional forwarding would have disappeared (cf. Fig. 3a). In our case of uniform traffic demand and bidirectional paths, this would have yielded the same solution as simple load balancing. The difference to single-directional forwarding would still have remained.

Multidirectional traffic gives more possibilities to choose such combinations of active links that the interfering transmitters are as far away from the receivers as possible. Hence, the difference between single- and multidirectional traffic should also be observable with more complex interference models where this is beneficial. However, a more detailed study remains as a topic for future research.

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