Flow-level performance of wireless data networks

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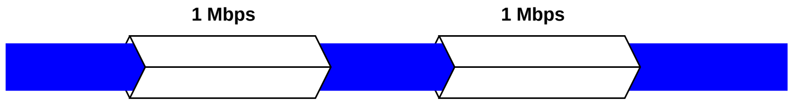
Outline

1. Flow-level model for wireless networks
2. Capacity sets of wireless networks
3. Flow-level performance analysis
4. Conclusions

1. Flow-level model for wireless networks

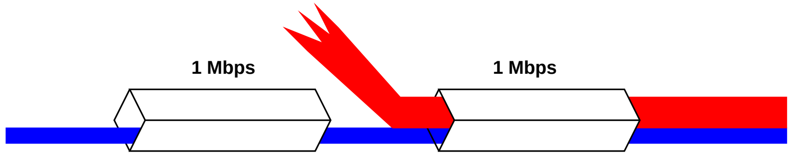
Flow-level modeling

- A flow is a file transfer from a source to a destination through a network
- During the transfer the network allocates the same capacity to the flow on all the links it traverses
- Performance measure for a flow: duration of the file transfer



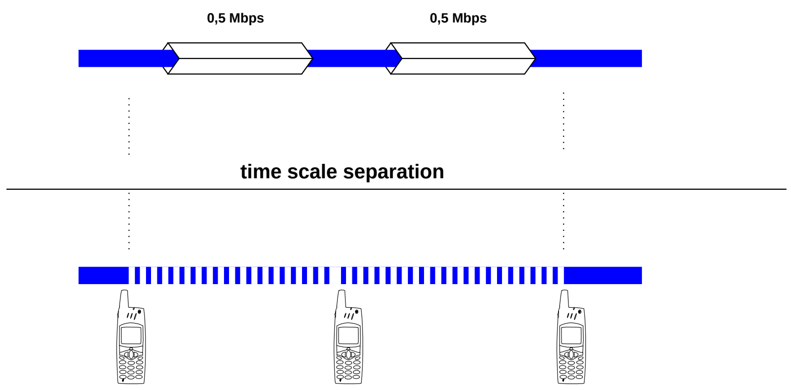
Flow-level modeling (2)

- The traffic is dynamic: file transfers start at random and depart upon completion
- During the transfer, the flows *share* the available capacity
- Performance of a flow depends on the traffic, available capacity and on how exactly the capacity is shared
- As the traffic is random, performance can be meaningfully described only as an expectation
- Flow throughput on route i: $\gamma_i = \mathrm{E}[S_i]/\mathrm{E}[T_i]$



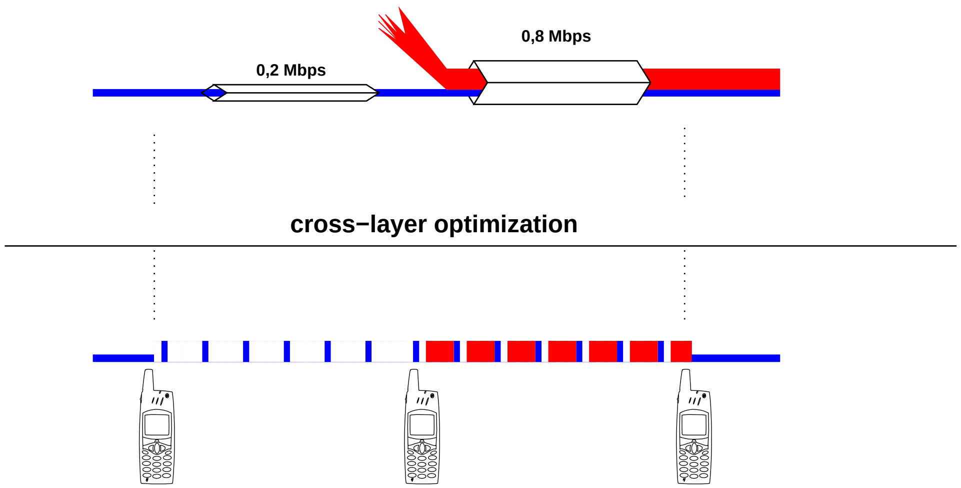
Flow-level modeling of wireless networks

- In wireless networks the transmission may interfere - not all links can be active simultaneously
  - access resolution required: scheduling, random access
- However, there is a strong time scale separation between access layer time slots and flow durations
  - from the flow-level point of view we may treat the wireless network *as if* it was a fixed network with link capacities that depend on MAC-layer functionality



Flow-level modeling of wireless networks (2)

- Capacity can be shifted from one link to another by scheduling
- Cross-layer optimization of resource allocation
- What is the flow-level performance of a wireless network when the MAC-layer is working ideally?



Performance analysis

- Flows share the available capacity using *balanced fairness* (BF)
- Capacity of the system is defined by the *capacity set* of the system
- The capacity set depends on the MAC-layer, two of which are considered here
  - centralized link scheduling
  - random access

Flow throughput under BF

$$\gamma_i = \frac{\mathrm{E}[S_i]}{\mathrm{E}[T_i]} = \frac{\rho_i}{\mathrm{E}[x_i]} = \frac{\rho_i}{\sum_{\mathbf{x}} x_i \pi(\mathbf{x})} = \frac{\rho_i}{\frac{\rho_i}{G(\boldsymbol{\rho})} \frac{\partial}{\partial \rho_i} G(\boldsymbol{\rho})} = \frac{G(\boldsymbol{\rho})}{\frac{\partial}{\partial \rho_i} G(\boldsymbol{\rho})} \,,$$

$$G(\boldsymbol{\rho}) = \sum_{\mathbf{x}} \Phi(\mathbf{x}) \rho_1^{x_1} \ldots \rho_N^{x_N} \,,$$

$$\Phi(\mathbf{x}) = \min\{\alpha : \frac{\tilde{\boldsymbol{\Phi}}(\mathbf{x})}{\alpha} \in \mathcal{C}\},$$

$$\tilde{\boldsymbol{\Phi}}(\mathbf{x}) = (\Phi(\mathbf{x} - \mathbf{e}_1), \ldots, \Phi(\mathbf{x} - \mathbf{e}_N))^{\mathrm{T}}.$$

2-step analysis

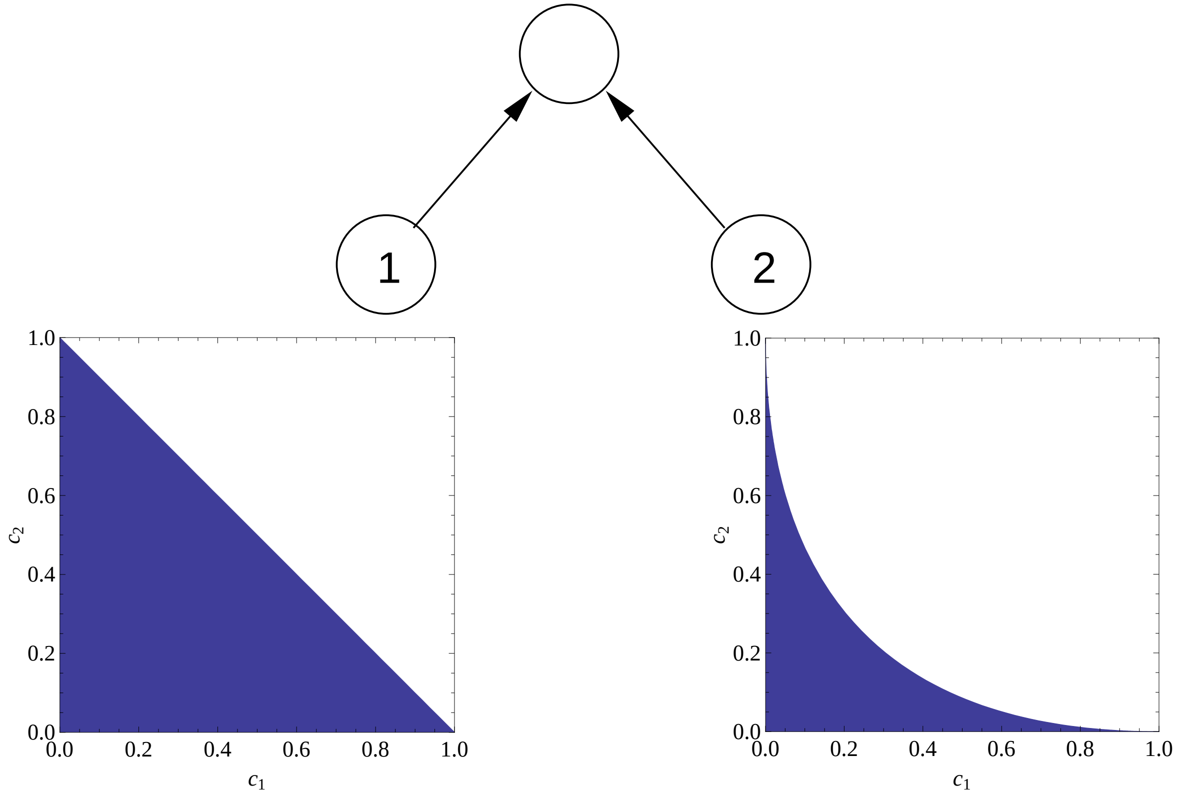
1. Determine the capacity set of the network
   - What is the maximum achievable capacity with given link capacity proportions?
2. Evaluate the flow throughput using the BF formulae

2. Capacity sets of wireless networks

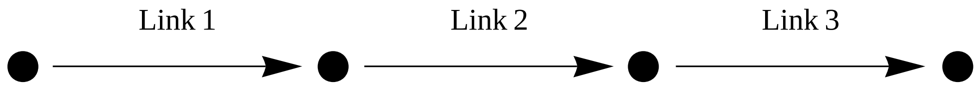
Capacity sets

- Capacity set: the set of flow-level link capacities that can be achieved by adjusting MAC-layer parameters
- Centralized link scheduling:
  - allocate time to sets of links that can be active simultaneously (transmission modes)
  - flow-level link capacity is the instantaneous link capacity averaged over the schedule
- Random access
  - determine link access probabilities for each link (the link is active with that probability in each time slot)
  - flow-level capacity is the probability of a non-conflicting transmission on the link

Example


1
2
1.0
0.8
0.6
0.4
0.2
0.0
c_2
0.0 0.2 0.4 0.6 0.8 1.0
c_1
1.0
0.8
0.6
0.4
0.2
0.0
c_2
0.0 0.2 0.4 0.6 0.8 1.0
c_1

Capacity sets under scheduling

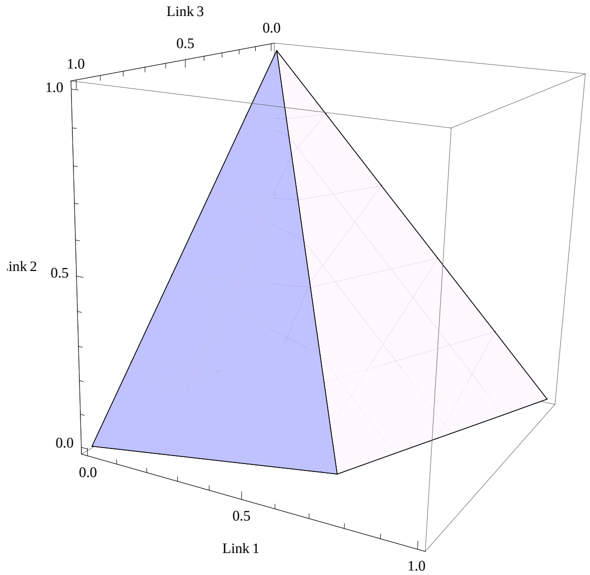


- Transmission modes

$$R = \begin{pmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{5} \end{pmatrix}$$

- Inequality constraints

$$\begin{aligned} \tfrac{3}{10}c_1 + c_2 + c_3 &\leq 1 \\ c_1 + c_2 + \tfrac{5}{12}c_3 &\leq 1 \end{aligned}$$



Capacity set of an Aloha network

- Let $\boldsymbol{\sigma}$ define the relative link capacities
- Find the maximum coefficient s so that $s\boldsymbol{\sigma}$ lies within the capacity set
- Consider first a single node i. The local view of the value s, denoted by s_i, solves

$$p_{ij} \prod_{k \in N_{ij}} (1 - P_k) = s_i \sigma_{ij}, \quad \forall\, j \in L_i$$

$$\sum_{j \in L_i} p_{ij} = P_i$$

- Thus, locally

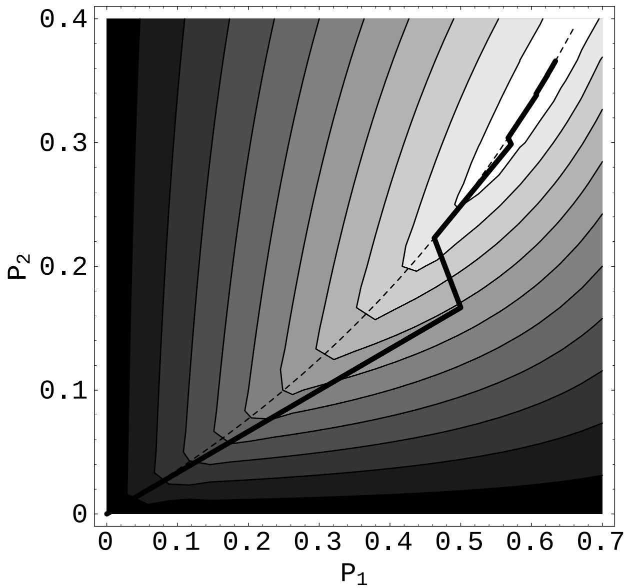
$$s_i = P_i \left(\sum_{j \in L_i} \frac{\sigma_{ij}}{\prod_{k \in N_{ij}} (1 - P_k)} \right)^{-1}$$

Capacity set of a multihop Aloha network

- Global optimization

$$s = \max_{\mathbf{P}} \min_{i} P_i \left(\sum_{j \in L_i} \frac{\sigma_{ij}}{\prod_{k \in N_{ij}} (1 - P_k)} \right)^{-1}$$

- We have developed an algorithm to solve the global problem



3. Flow-level performance analysis

BF analysis: The ideal case

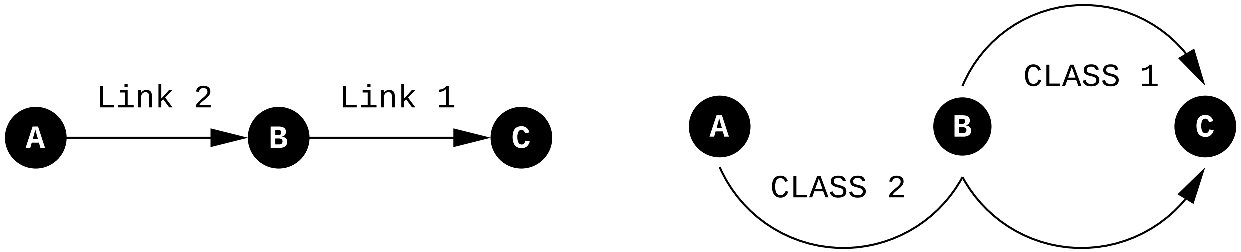
- In simple networks the analysis yields analytical expressions for the throughput
  - if you can compute the normalization constant

$$G(\boldsymbol{\rho}) = \sum_{\mathbf{x}} \Phi(\mathbf{x}) \rho_1^{x_1} \ldots \rho_N^{x_N},$$

  - the throughput follows:

$$\gamma_i = \frac{G(\boldsymbol{\rho})}{\frac{\partial}{\partial \rho_i} G(\boldsymbol{\rho})}$$

Example: Analysis of a two-hop wireless network



- Two classes (routes) and two links that cannot be active simultaneously

Example: Analysis of a two-hop wireless network

$$C_1(p) = p, \qquad C_2(p) = 1 - p$$

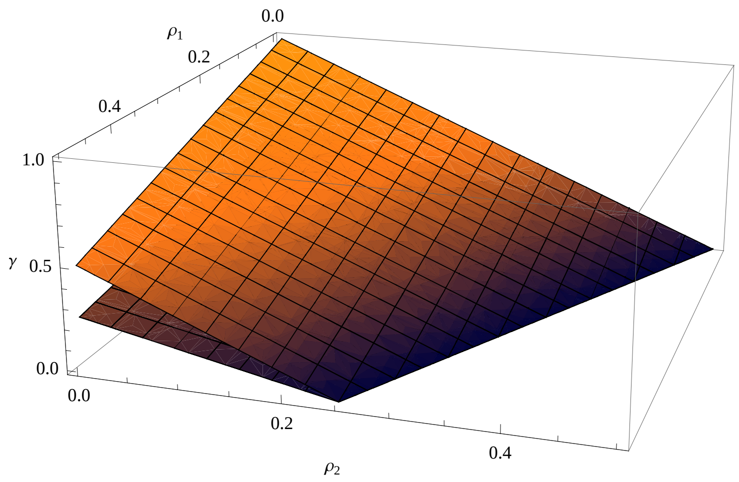
$$\Phi(x) = \min_p \max\{\tfrac{\Phi(x-e_1)+\Phi(x-e_2)}{p}, \tfrac{\Phi(x-e_2)}{1-p}\}$$

$$\Phi(x) = \Phi(x - e_1) + 2\,\Phi(x - e_2)$$

$$\Phi(x) = \binom{x_1+x_2}{x_1} 2^{x_2}$$

$$G(\rho) = \frac{1}{1-\rho_1-2\rho_2}$$

$$\gamma_1 = 1-\rho_1-2\rho_2, \quad \gamma_2 = \frac{1}{2}(1-\rho_1-2\rho_2)$$



Example: Analysis of a two-user, single resource Aloha

- Consider a single channel system shared by 2 users. It can be shown that the capacity set is defined by

$$\sqrt{c_1} + \sqrt{c_2} = 1$$

- It can be shown that the balance function of this system is obtained simply by squaring the balance function of a normal processor sharing system

- Furthermore, it turns out the normalization constant can be computed

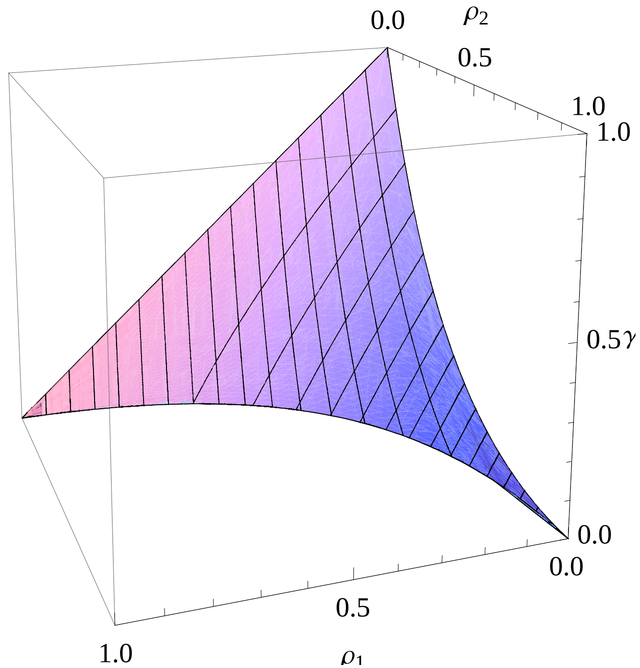
$$G_2(\rho_1, \rho_2) = \sum_{x_1, x_2} \binom{x_1 + x_2}{x_1,\ x_2}^2 \rho_1^{x_1} \rho_2^{x_2} = \frac{1}{\sqrt{1 - 2(\rho_1 + \rho_2) + (\rho_1 - \rho_2)^2}}.$$

Example: Analysis of a two-user, single resource Aloha (2)

- The throughput

$$\gamma_i = \frac{1 - 2(\rho_1 + \rho_2) + (\rho_1 - \rho_2)^2}{1 \mp (\rho_1 - \rho_2)}$$

- Throughput of class 1 as a function of the loads



BF analysis: The general case

- Generally, the recursion does not have a closed form solution
- In addition, as the number of routes increase it quickly becomes intractable to compute the recursion numerically
- Approximate analysis possible based on asymptotic throughput behavior
  - study the throughput with a fixed traffic profile
  - compute the values and derivatives of the throughput at zero load and at the capacity limit
  - use interpolation to sketch the throughput in-between

Asymptotic analysis: Light traffic

- Light traffic throughput

$$\begin{cases} \gamma_i(0) &= \dfrac{G(0)}{G_i(0)} \\ \gamma_i'(0) &= \dfrac{G_i(0)G'(0) - G(0)G_i'(0)}{G_i(0)^2} \\ \gamma_i''(0) &= \dfrac{2G_i'(0)\big(G(0)G_i'(0) - G'(0)G_i(0)\big) + G_i(0)\big(G_i(0)G''(0) - G(0)G_i''(0)\big)}{G_i(0)^3} \end{cases}$$

Asymptotic analysis: Light traffic (2)

- Notation: $G(0) = 1$ and

$$G'(0) = \sum_{j=1}^{N} \Phi(\mathbf{e}_j)\hat{\rho}_j, \qquad G_i(0) = \Phi(\mathbf{e}_i),$$

$$G''(0) = 2\sum_{j=1}^{N}\sum_{k=j}^{N} \Phi(\mathbf{e}_j + \mathbf{e}_k)\hat{\rho}_j\hat{\rho}_k, \qquad G_i'(0) = \sum_{j=1}^{N} \Phi(\mathbf{e}_i + \mathbf{e}_j)\hat{\rho}_j + \Phi(2\mathbf{e}_i)\hat{\rho}_i,$$

$$G_i''(0) = 2\left(\sum_{j=1}^{N}\sum_{k=j}^{N} \Phi(\mathbf{e}_i + \mathbf{e}_j + \mathbf{e}_k)\hat{\rho}_j\hat{\rho}_k + \sum_{j=1}^{N} \Phi(2\mathbf{e}_i + \mathbf{e}_j)\hat{\rho}_i\hat{\rho}_j + \Phi(3\mathbf{e}_i)\hat{\rho}_i^2\right).$$

- Where the balance function is recursively defined as

$$\hat{\boldsymbol{\rho}} = \frac{\mathbf{p}}{\Upsilon(\mathbf{Ap})}, \qquad \Phi(\mathbf{x}) = \Upsilon(\mathbf{A}\tilde{\boldsymbol{\Phi}}(\mathbf{x})), \qquad \begin{aligned} \Upsilon(\mathbf{b}) = \min_{\mathbf{q}} \quad & \mathbf{e}^{\mathrm{T}}\mathbf{q}, \\ \text{s.t.} \quad \mathbf{Rq} & \geq \mathbf{b}, \\ \mathbf{q} & \geq 0. \end{aligned}$$

Asymptotic analysis: Heavy traffic

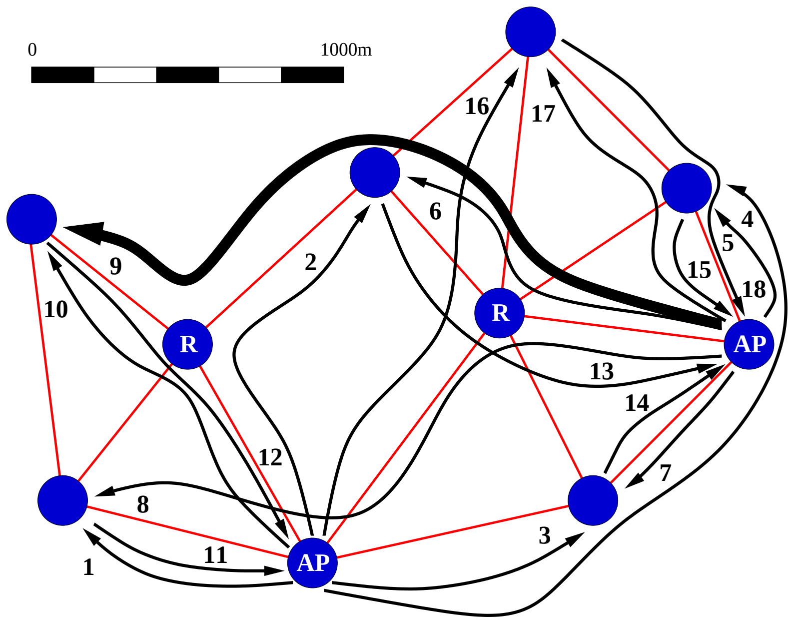
- At heavy load any saturated flow class throughput goes to zero
- The heavy load derivative depends only on the *saturated* constraints.
- In wireless networks one enumerates all the spanning vectors of the solution space of

$$\begin{aligned} &\max_{\mathbf{u}} \ \mathbf{u}^{\mathrm{T}}\mathbf{A}\mathbf{p}, \\ \text{s.t.} \quad \mathbf{R}^{\mathrm{T}}\mathbf{u} &\leq \mathbf{e}, \\ \mathbf{u} &\geq \mathbf{0}. \end{aligned}$$

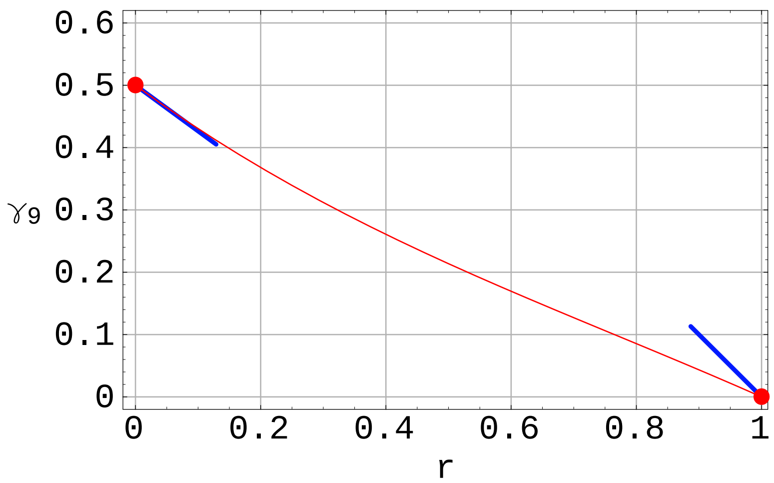
- Finally, the derivative is given by

$$\gamma_i'(1) = -\frac{1}{\sum_j \mathbf{a}_i^{\mathrm{T}}\mathbf{u}_j}.$$

Example



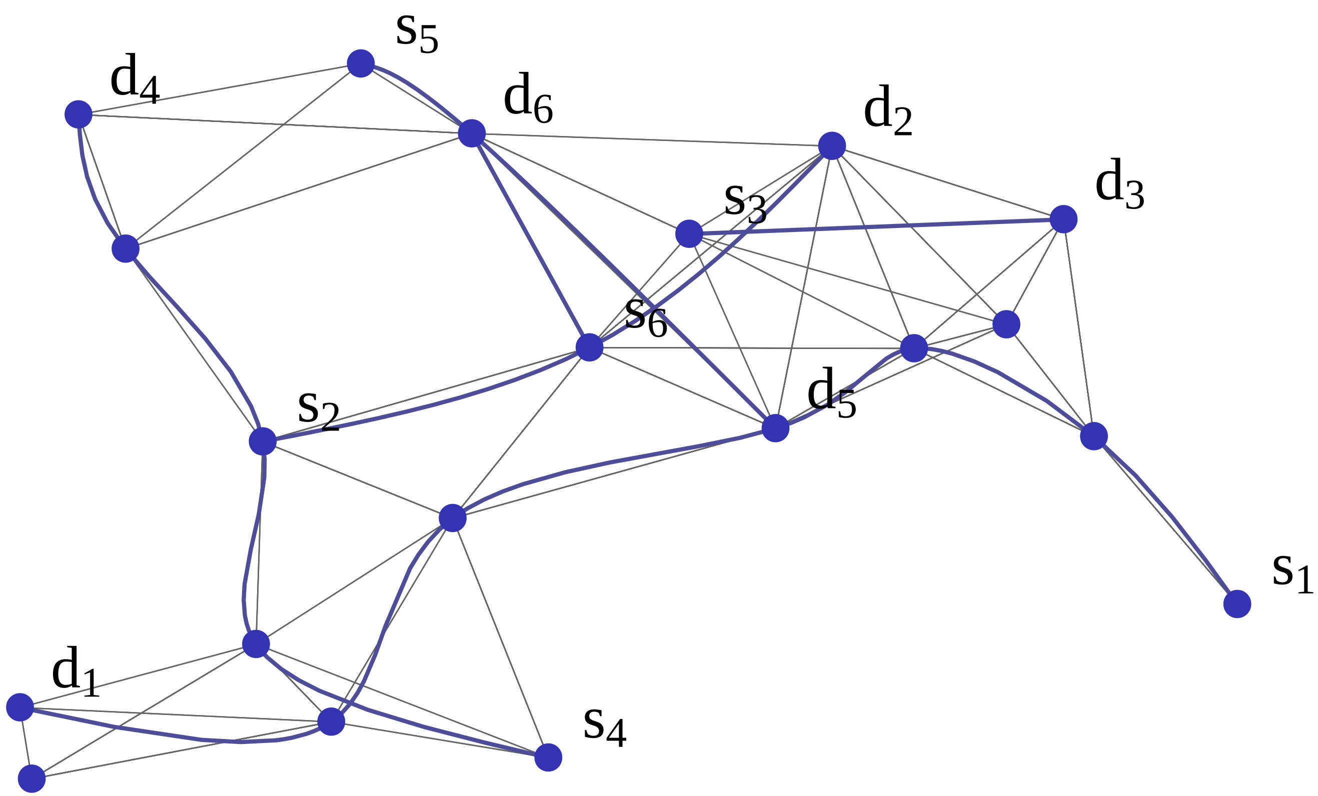
0
1000m
R
R
AP
AP
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18



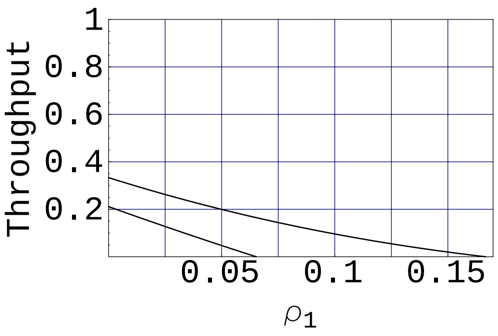
γ_9
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
r

Example: Multihop network

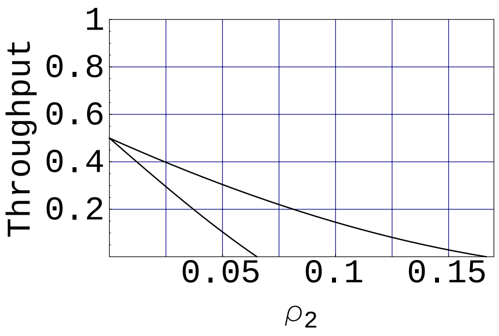
- Consider the following multihop network with 20 nodes and 6 routes



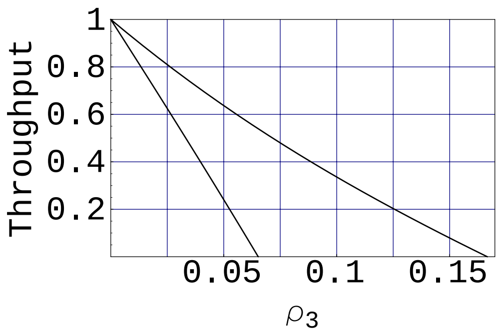
Example: Multihop network (2)



(a) Class 1



(b) Class 2



(c) Class 3

Figure 1: Asymptotic analysis of traffic classes 1, 2 and 3. The upper curves correspond to coordinated MAC and the lower curves to Aloha.

4. Conclusions

Conclusions

- Performance analysis of a wireless network:
  - Input:
    - network topology and routing
    - traffic load (bps) defined for each route
  - Output:
    - expected flow throughput on a given route
- Procedure:
  - define capacity set
  - carry out BF analysis
    - analytical approach for simple cases
    - generally asymptotic approximation or throughput bounds