Dimensioning of Data Networks

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Outline

1. The data network dimensioning problem
2. Dimensioning of fixed networks
3. Extensions
4. Dimensioning of wireless mesh networks
5. Conclusions

1. The data network dimensioning problem

The dimensioning problem

- Consider a communications network carrying *elastic* traffic, e.g., file transfers, on pre-defined routes
- How much capacity you must allocate on each link to meet the performance requirements of data traffic?

The dimensioning problem (2)

- Input parameters:
  - network topology and routing for each traffic class
  - link capacity costs
  - traffic load (bps) of each class (note that the system is dynamic, i.e. the file transfers are initiated randomly and depart upon completion)
  - performance requirement for each route of for the average over the routes
- Output:
  - link capacities (fixed networks)
  - radio capacity (wireless networks)

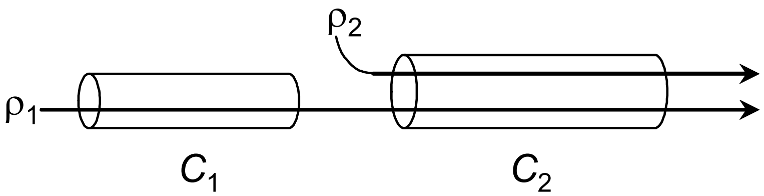
The dimensioning problem (3)

- We define the throughput of a flow class (route) i, denoted by γ_i as the ratio of mean flow (file) size and flow duration in the class
- Assume that the bandwidth is allocated according to the balanced fairness principle, then

$$\left(\sum_{l \in \mathcal{R}_i} \frac{1}{c_l - r_l}\right)^{-1} \leq \gamma_i \leq \min_{l \in \mathcal{R}_i} c_l - r_l$$

  where c_l and $r_l = \sum_{i \in \mathcal{F}_l} \rho_i$ are the capacity and load of link l, respectively
- Since the link capacity must be larger than the offered load, we need to determine the excess capacities $d_l = c_l - r_l$

Example



Capacity costs of the links equal,

$$\gamma_i = 1\text{Mbps},$$

$\rho_1 = 5$Mbps, $\rho_2 = 7$Mbps,

C_1, C_2 ?

2. Dimensioning of fixed networks

Average throughput constraint

- Consider only the weighted average throughput of the network:

$$\min_{\mathbf{d}} \mathbf{w}^{\mathrm{T}}\mathbf{d},$$
$$\left(\sum_i \frac{\rho_i}{\rho_{\mathrm{tot}}} \sum_{l\in\mathcal{R}_i} \frac{1}{d_l}\right)^{-1} \geq \gamma_{\mathsf{ave}},$$
$$\mathbf{d} > 0$$

- Parameters: $\mathbf{w}$ link costs, $\mathbf{d}$ excess capacities
- This yields explicit values for the excess capacities

$$d_l = \gamma_{\mathsf{ave}} \sqrt{\frac{r_l}{w_l \rho_{\mathrm{tot}}}} \sum_j \sqrt{\frac{r_j w_j}{\rho_{\mathrm{tot}}}} \; \forall l.$$

Per-class constraints

- The second model assumes that each route has its own throughput target

$$\min_{\mathbf{d}} \mathbf{w}^{\mathrm{T}}\mathbf{d},$$
$$\Gamma_i(\boldsymbol{\rho}, \mathbf{c}) \geq \gamma_i, \quad \forall i$$
$$\mathbf{d} > 0$$

- We have two approaches based on the limit expressions for throughput under balanced fairness

Per-class constraints: Lower bound

- Problem formulation

$$\min_{\mathbf{d}} \mathbf{w}^{\mathrm{T}}\mathbf{d},$$
$$\min_{l\in\mathcal{R}_i} d_l \geq \gamma_i, \quad \forall i$$
$$\mathbf{d} > 0$$

- Excess capacity must be at least as large as the largest throughput demand on that link:

$$d_l = \max_{i\in\mathcal{F}_l} \gamma_i$$

Per-class constraints: Upper bound

- Dimensioning problem with store-and-forward bound as the throughput model

$$\min_{\mathbf{d}} \mathbf{w}^{\mathrm{T}}\mathbf{d}$$

$$\mathbf{A}\mathbf{d}^{-1} \leq \boldsymbol{\gamma}^{-1},$$
$$\mathbf{d} > 0$$

- Parameters: $\mathbf{A}$ route-link incidence matrix (routing), $\boldsymbol{\gamma}$ throughput demands

Solution

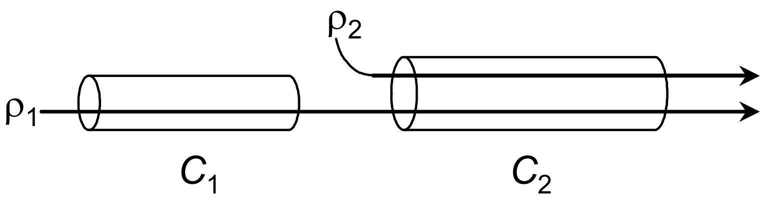
- Iterative algorithm for dual variables

$$\mathbf{u}_k = \mathbf{u}_{k-1} * \boldsymbol{\gamma} * \mathbf{A}\sqrt{\frac{\mathbf{w}}{\mathbf{A}^{\mathrm{T}}\mathbf{u}_{k-1}}}$$

- It can be shown that the iteration converges from any strictly positive $\mathbf{u}_0$ to a unique $\mathbf{u}_*$
- Primal solution

$$\mathbf{d} = \sqrt{\frac{\mathbf{A}^{\mathrm{T}}\mathbf{u}_*}{\mathbf{w}}}$$

Example



$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

$$\gamma_i = 1\text{Mbps},$$

$$\rho_1 = 5\text{Mbps}, \quad \rho_2 = 7\text{Mbps}.$$

$$\mathbf{u}_k = \mathbf{u}_{k-1} * \boldsymbol{\gamma} * \mathbf{A}\sqrt{\frac{\mathbf{w}}{\mathbf{A}^{\mathrm{T}}\mathbf{u}_{k-1}}}.$$

Iter.	u_1	u_2
1	1.00	1.00
2	1.71	0.71
3	2.41	0.46
4	2.97	0.27
5	3.38	0.15
10	3.97	0.01
20	4.00	0.00

- $\mathbf{u}_* = (4,\ 0)^{\mathrm{T}}$.
- $\mathbf{d} = (2,\ 2)^{\mathrm{T}}$.
- $C_1 = \rho_1 + d_1 = 7$Mbps
- $C_2 = \rho_1 + \rho_2 + d_2 = 14$Mbps.

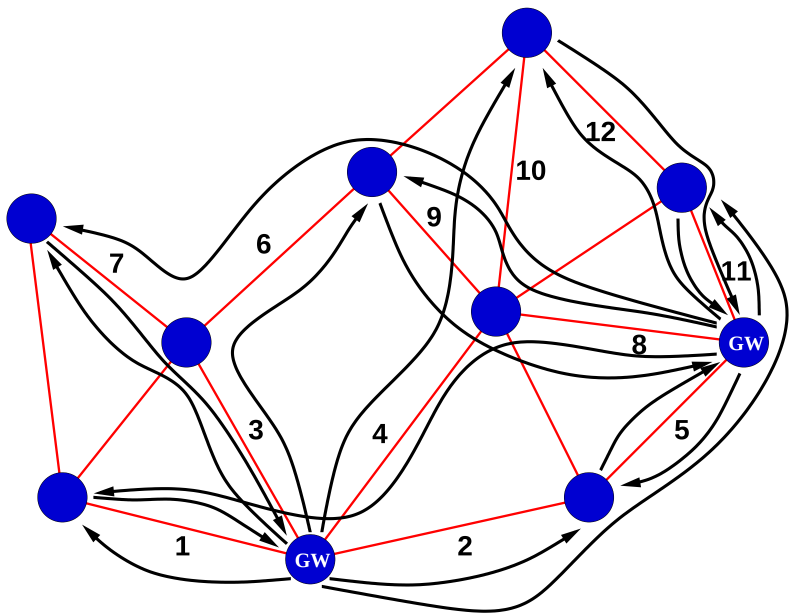
Per-class constraints (3): Alternative upper bound

- "Improved store-and-forward bound":

$$\gamma_i \leq \left(\max_{l \in \mathcal{R}_i} \frac{1}{d_l + r_l} + \sum_{l \in \mathcal{R}_i} \frac{r_l}{d_l(d_l + r_l)} \right)^{-1}$$

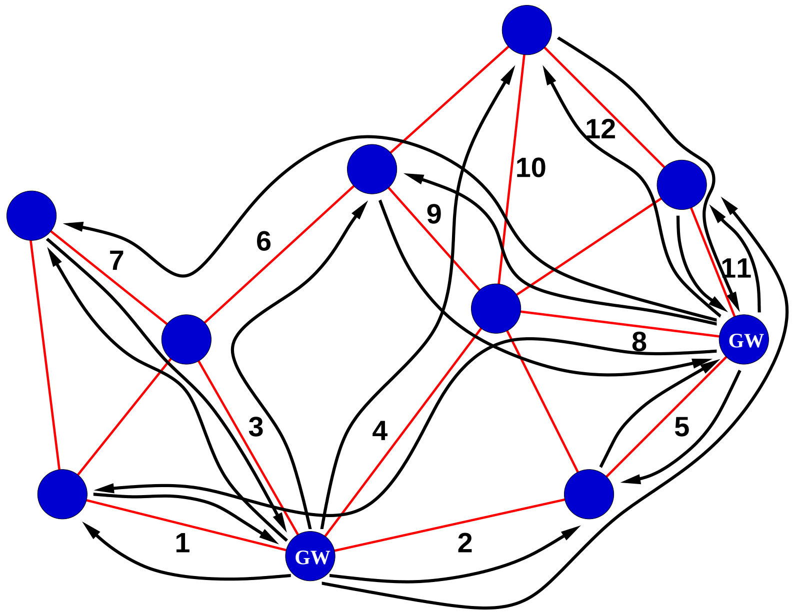
- Can be solved analogously to the SF case
- Increased computational complexity:
  - for each flow class, the number of constraints equals to the number of links used

Example



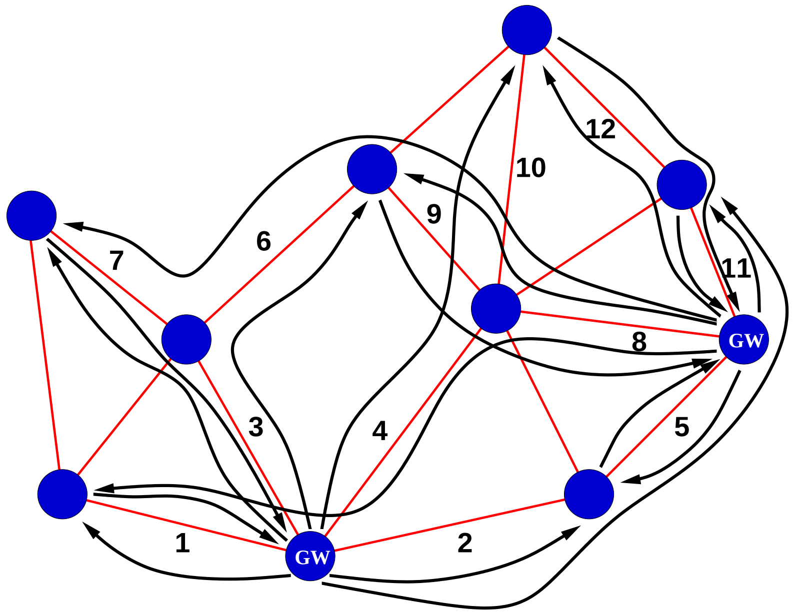
- 18 flow classes, 12 links
- w_l=1 for all l
- $\rho_i = 1$ Mbps for all i
- γ_{ave}= 0.1 Mbps, γ_i= 0.1 Mbps

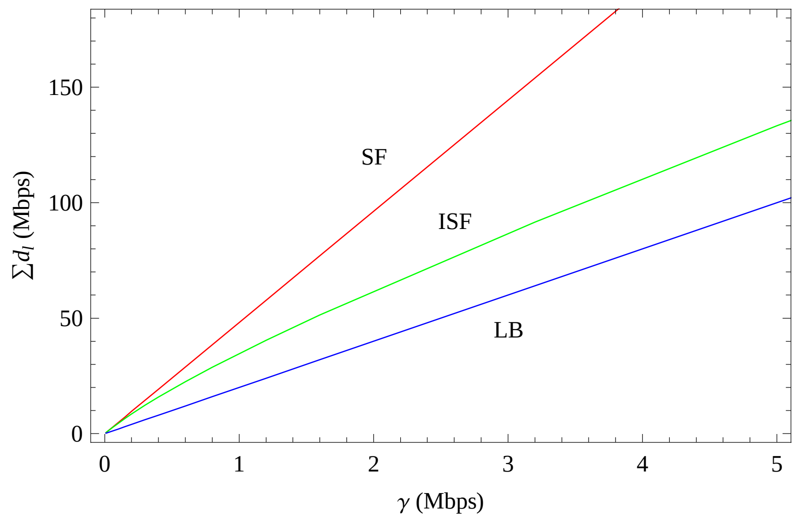
Example (2)



Link	AVE	LB	UB-SF	UB-ISF
1	3.19	3.1	3.27	3.23
2	2.15	2.1	2.29	2.28
3	3.19	3.1	3.13	3.13
4	2.15	2.1	2.28	2.27
5	3.19	3.1	3.29	3.27
6	2.15	2.1	2.39	2.36
7	3.19	3.1	3.39	3.36
8	4.22	4.1	4.45	4.41
9	3.19	3.1	3.38	3.35
10	1.11	1.1	1.15	1.15
11	5.24	5.1	5.32	5.31
12	2.15	2.1	2.14	2.14

Example (3)

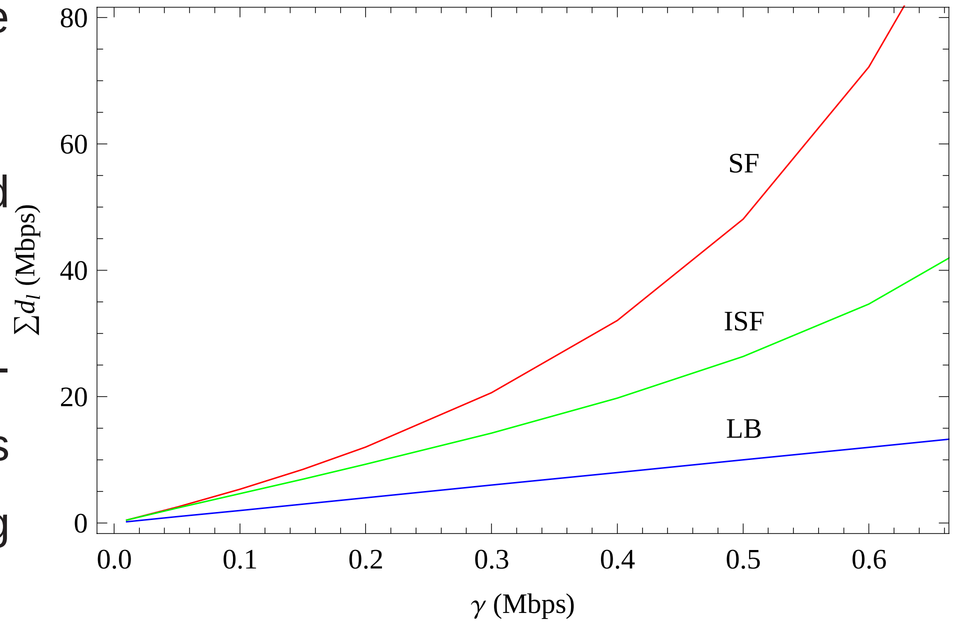

GW
GW
1
2
3
4
5
6
7
8
9
10
11
12


SF
ISF
LB
$\sum d_l$ (Mbps)
γ (Mbps)
0
50
100
150
0
1
2
3
4
5

3. Extensions

Effects of access rate limitations

- Flow access rates may have per-flow or per-class limits
- These can be easily included in the algorithms
- When the throughput requirement approaches the access rate limit, the dimensioning costs increase considerably



Multiservice networks

- We have considered dimensioning of data networks subject to constraints on file transfer performance
- Also delay-constrained real-time traffic could be included (e.g. VoIP calls)
- In M/M/1 queuing network the packet delay on route i is

$$\mathrm{D}_i(\mathbf{c}) := \sum_{l \in \mathcal{R}_i} \frac{1}{c_l - r_l} \leq T_i$$

- Add delay constraints to the formulation

Multiservice networks (2)

- Three approaches proposed: Best effort network, separate networks, and prioritized real-time traffic

$$\min_{\mathbf{d}} \mathbf{w}^{\mathrm{T}}\mathbf{d}$$
$$\mathbf{A}^{\mathrm{el}}\mathbf{d}^{-1} \leq \boldsymbol{\gamma}^{-1},$$
$$\mathbf{A}^{\mathrm{rt}}\mathbf{d}^{-1} \leq \boldsymbol{t},$$
$$\mathbf{d} > 0$$

$$\min_{\mathbf{c}^{\mathrm{el}}} \mathbf{w}^{\mathrm{T}}\mathbf{c}^{\mathrm{el}},$$
$$\Gamma_i(\mathbf{c}^{\mathrm{el}}) \geq \gamma_i, \quad \forall\, i,$$
$$\mathbf{c}^{\mathrm{el}} > \mathbf{r}^{\mathrm{el}}$$

$$\min_{\mathbf{c}^{\mathrm{rt}}} \mathbf{w}^{\mathrm{T}}\mathbf{c}^{\mathrm{rt}},$$
$$\mathrm{D}_i(\mathbf{c}^{\mathrm{rt}}) \leq T_i, \quad \forall\, i,$$
$$\mathbf{c}^{\mathrm{rt}} > \mathbf{r}^{\mathrm{rt}}$$

$$\min_{\mathbf{c}} \mathbf{w}^{\mathrm{T}}\mathbf{c},$$
$$\mathrm{D}_i(\mathbf{c}, \mathbf{r}^{\mathrm{rt}}) \leq T_i, \quad \forall\, i,$$
$$\Gamma_i(\mathbf{c} - \mathbf{r}^{\mathrm{rt}}, \mathbf{r}^{\mathrm{el}}) \geq \gamma_i, \quad \forall\, i,$$
$$\mathbf{c} > \mathbf{r}$$

4. Dimensioning of wireless mesh networks

Dimensioning of wireless networks

- Obtain an order-of-magnitude estimate of the required physical resources of a wireless mesh network when
  - the performance target is expressed in terms of average flow throughput for data traffic
  - the capacity of network is efficiently utilized by scheduling
- Assumption: all nodes use only one transmission rate (or other radio parameter) which is to be dimensioned

Dimensioning of mesh networks: Scheduling

- Virtual link capacities are obtained by scheduling:

$$\mathbf{c} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & c_{15} & 0 & c_{17} \\ 0 & c_{22} & 0 & 0 & 0 & c_{26} & 0 \\ 0 & 0 & c_{33} & 0 & c_{35} & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & c_{46} & c_{47} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \end{pmatrix}$$

where $\mathbf{t}$ is the schedule satisfying $\sum_i t_i = 1$

Dimensioning of mesh networks: Capacity parameterization

- For nominal link rate dimensioning

$$\mathbf{c} = b\mathbf{M}\mathbf{t}$$

where $m_{lj} = 1$ if link l is active in transmission mode j and 0 otherwise.

- For nominal tranmission power dimensioning

$$\mathbf{c} = \mathbf{S}(p)\mathbf{t}$$

where s_{lj} is the Shannon capacity of link l when it is interfered by the other active links in transmission mode j

  – transmission power is fixed to p for all active links

Dimensioning of mesh networks: Throughput

- Generally, route throughputs of the dynamic system cannot be expressed in a closed form (even if balanced resource allocation is assumed)
- We may utilize different bounds for performance under balanced fairness
  - Store-and-forward bound:

$$\Gamma_i(\mathbf{c}) = \left(\sum_{l \in \mathcal{R}_i} d_l^{-1} \right)^{-1}$$

  - Upper bound for throughput:

$$\Gamma_i(\mathbf{c}) = \left(\max_{l \in \mathcal{R}_i} \frac{1}{c_l - r_l} \right)^{-1}$$

Dimensioning of mesh networks: Problem

- Problem formulations for both capacity parameters:

$$
\begin{aligned}
b &= \min_{\mathbf{q}} \mathbf{e}^{\mathrm{T}}\mathbf{q}, \\
& \Gamma_i(\mathbf{M}\mathbf{q}) \geq \gamma_i, \quad \forall i, \\
& \mathbf{q} \geq 0
\end{aligned}
\qquad
\begin{aligned}
p &= \{p : \min_{\mathbf{q}} \mathbf{e}^{\mathrm{T}}\mathbf{q} = 1\}, \\
& \Gamma_i(\mathbf{S}(p)\mathbf{q}) \geq \gamma_i, \quad \forall i, \\
& \mathbf{q} \geq 0
\end{aligned}
$$

- Difficult computational problem:
  - number of variables large (number of modes large)
  - difficult non-linear constraints
- Approximative methods are needed

Approximation I: LP

- The upper bound of throughput

$$\Gamma_i(\mathbf{c}) = \left(\max_{l \in \mathcal{R}_i} \frac{1}{c_l - r_l} \right)^{-1}, \forall i \qquad \Leftrightarrow \qquad c_l \geq r_l + \varphi_l, \forall l$$

where $\varphi_l = \max_{a_{li}=1} \gamma_i$

- Now the scheduling part reduces to an LP problem:

$$\begin{aligned} b &= \min_{\mathbf{q}} \mathbf{e}^{\mathrm{T}} \mathbf{q}, \\ & \mathbf{M}\mathbf{q} \geq \mathbf{r} + \boldsymbol{\varphi}, \\ & \mathbf{q} \geq 0 \end{aligned}$$

Approximation II: Fixed schedule (FS)

- Idea:
  - Fix the schedule $\mathbf{t}^*$ by, e.g., solving the LP approximation
  - Scale the capacity parameter upwards until the improved store-and-forward bound is satisfied for each route
- Now we have a simple line search:

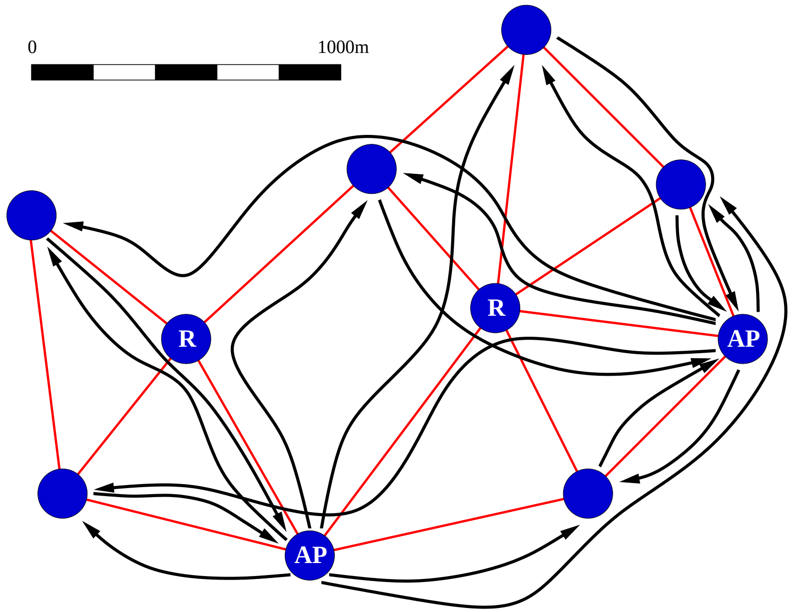
$$
\begin{aligned}
b &= \arg\min_{b} \\
& \Gamma_i(b\mathbf{M}\mathbf{t}^*) \geq \gamma_i, \quad \forall i
\end{aligned}
$$

Approximation III: Fixed capacity vector

- Solve the virtual link capacities $\mathbf{c}^*$ by using a fixed network dimensioning formulation
- Again, the scheduling part becomes an LP problem:

$$\begin{aligned} b &= \min_{\mathbf{q}} \mathbf{e}^{\mathrm{T}}\mathbf{q}, \\ & \mathbf{M}\mathbf{q} \geq \mathbf{c}^*, \\ & \mathbf{q} \geq 0 \end{aligned}$$

Numerical example



- 22 links, 18 traffic classes (routes)
- $\rho_i = 2$ Mbps, $\gamma_i = 0.2$ Mbps
- (Radio parameters)
- Capacity limits b_{CL}=36 Mbps, p_{CL}=350 mW

Numerical example: Results

Approximation	Boolean (Mbps)	Shannon (mW)
LP	38.2	500
FS	41.1	1390
FC	41.8	860

5. Conclusions

Conclusions

- Goal:
  - to provide order-of-magnitude estimates of required resources,
  - to evaluate feasibility of a given network plan.
- Pros
  - easy-to-obtain motivated quantitative results
  - only load information suffices on traffic, no need for distribution
- Cons
  - fixed routing
  - resource sharing may differ in practice
- First step in dimensioning process