

NAME

Qmd1(), Qnnd1(), Qsdd1() – Virtual waiting time distribution functions

SYNOPSIS

```
#include <queue.h>

double Qmd1(double x, double rho);
double Qnnd1(double x, int N, double D);
double Qsdd1(double x, double *D, long N);
```

DESCRIPTION

delim \$\$ These functions return the virtual waiting time distribution for different queuing models. Parameter x is the amount of unfinished work in the system.

Qmd1() is a model for the M/D/1 queuing system with Poisson arrivals and deterministic (constant) service time. ρ is the load level of the system.

Qnnd1() is the N*D/D/1 queuing system which has constant service time and N deterministic sources with the same period D , so that the load level of system is N/D .

Qsdd1() is the $\sum D_{sub \{i\}} / D/1$ queuing model for a system with number of deterministic sources N , each having its own period, and a constant service time. Table of periods is given by D .

ALGORITHM

M/D/1 waiting time distribution is calculated using three different algorithms:

When $\rho < 0.3$ and $x < (9 + 15 * \text{Log10}(0.3 / \rho))$ the upper limit formula:

$$\sum_{n=x}^{\infty} \{ \{ (\rho (n-x)) \sup \{ n \} \text{ over } n! \} e^{\sup \{ -\rho (n-x) \}} (1 - \rho) \} \}$$

is used. Terms are calculated logarithmically to avoid overflow.

If $\rho < 0.3$ and $x > (9 + 15 * \text{Log10}(0.3 / \rho))$ or $\rho > 0.3$ and $x > 8$, $Q(x)$ is approximated by

$$\{ C_{sub 0} \} \{ e^{\sup \{ -\{ r_{sub 0} \} x \}} \}$$
, where

$$\{ C_{sub 0} \} = \{ 1 - \rho \} \text{ over } \{ \rho \{ e^{\sup \{ r_{sub 0} \}} - 1 \} \}$$
 and $\{ r_{sub 0} \}$ is solved from

$$\rho \{ \{ e^{\sup \{ r_{sub 0} \}} - 1 \} - \{ r_{sub 0} \} = 0 \}$$

Otherwise if $\rho > .3$ and $x < 8$, Q is calculated with the upper limit sum using an improved algorithm.

N*D/D/1 waiting time distribution is calculated using the following formula:

$$\{ Q_{sub X \sup N} \} (x) = \sum_{\{ x < n \leq N \}} \left(\{ \text{pile } \{ N \text{ above } n \} \} \text{ right} \right) \sim \left(\{ n - x \} \text{ over } D \text{ right} \right) \sup n \sim \left(1 - \{ n - x \} \text{ over } D \text{ right} \right) \sup \{ N - n \} \sim \{ D - N + x \} \text{ over } \{ D - n + x \}$$

Since the binomials in the formula would get very large, calculation is done by adding the logarithms of each term. These logarithms can be easily derived from previous terms.

$\sum D_{sub \{i\}} / D/1$ waiting time distribution is given by formula

$$\{ Q(x) \} \sim \sum_{\{ n > x \}} \{ \{ \psi (z_{sub n}) \} \text{ over } \{ z_{sub n} \sup n - d \} \sim 1 \text{ over } \{ \sqrt{2 \pi} \} \sigma (z_{sub n}) \} \sim \left(1 - \sum_{\{ j=1 \} \text{ to } N} \{ \rho_{sub j} \} \text{ over } \{ 1 - p_{sub j} + p_{sub j} z_{sub n} \} \right) \text{ right} \}$$

Values of $\{ z_{sub n} \}$ are determined from

$$\sum_{\{ i \}} \{ p_{sub i} z_{sub n} \} \text{ over } \{ 1 - p_{sub i} + p_{sub i} z_{sub n} \} = n - d$$

An approximating function is used to find the value of z .

ERRORS

When ρ is close to 1, **Qmd1()** might give inaccurate results.

SEE ALSO

COST 224: Performance evaluation and design of multiservice networks