

**NAME**

Fend1() – Virtual waiting time distribution function

**SYNOPSIS in a C-program**

```
#include <queue.h>
```

```
double Fend1(double x, int n, double rho);
```

**SYNOPSIS in Mathematica**

```
Fend1[Real, Integer, Real]
```

**SYNOPSIS with MathLink**

```
LnkFend1[Real, Integer, Real]
```

**DESCRIPTION**

delim \$\$ **Fend1**() is a model for the  $\{E \text{ sub } n\}/D/1$  queuing system with Erlang- $n$  arrivals and deterministic (constant) service time. Parameter  $x$  is the amount of unfinished work in the system and  $Rho$  is the load level of the system.

**ALGORITHM**

$\{E \text{ sub } n\}/D/1$  waiting time distribution is calculated using the following algorithm:

$$P\{SW \leq x\} = n \{F \text{ sub } n\}(x) = n \{e \sup \{\{\lambda\}x\}\{P \text{ sub } mn+n-1\}(x-m)\}.$$

To calculate this probability, we need to resolve the coefficients of polynomials  $\{P \text{ sub } k\}$  up to  $k = nm+n-1$ , where  $m$  is the integral part of  $x$ .

$$\{P \text{ sub } mn\} = (\{\sum\} \text{ from } \{j=0\} \text{ to } \{mn-n\} \{a \text{ sub } j \sup (mn-n)\}, \{-\beta\} \text{ over } 1\} \{a \text{ sub } 0 \sup (mn-1)\}, \{-\beta\} \text{ over } 2\} \{a \text{ sub } 1 \sup (mn-1)\}, \dots, \{-\beta\} \text{ over } mn\} \{a \text{ sub } mn-1 \sup (mn-1)\}),$$

$$\{P \text{ sub } mn+i\} = (\{\sum\} \text{ from } \{j=0\} \text{ to } \{mn+i-n\} \{a \text{ sub } j \sup (mn+i-n)\}, \{-\lambda\} \text{ over } 1\} \{a \text{ sub } 0 \sup (mn+i-1)\}, \{-\lambda\} \text{ over } 2\} \{a \text{ sub } 1 \sup (mn+i-1)\}, \dots, \{-\lambda\} \text{ over } mn+1\} \{a \text{ sub } mn+i-1 \sup (mn+i-1)\}),$$

where  $i=1, \dots, n-1$  and  $m=1, 2, \dots$

The recursion starts from the initial values  $\{a \text{ sub } 0 \sup (i-1)\} = \{P \text{ sub } i \sup o\}$ :

$$\{P \text{ sub } 0\} = (\{P \text{ sub } 1 \sup o\}),$$

$$\{P \text{ sub } i\} = (\{P \text{ sub } i+1 \sup o\}, \{-\lambda\} \text{ over } 1\} \{a \text{ sub } 0 \sup (i-1)\}, \{-\lambda\} \text{ over } 2\} \{a \text{ sub } 1 \sup (i-1)\}, \dots, \{-\lambda\} \text{ over } 3\} \{a \text{ sub } i-1 \sup (i-1)\}), \quad i=1, \dots, n-1.$$

What remains is to determine the initial values  $\{P \text{ sub } i \sup o\}$ .

**ERRORS**

When  $\rho$  is close to 1, these functions might give inaccurate results.

**SEE ALSO**

S. Aalto & J. Virtamo: M/D/n queue revisited.