

NAME

Qmd1(), Qnnd1(), Qsdd1() – Virtual waiting time distribution functions

SYNOPSIS

```
#include <queue.h>

double Qmd1(double x, double rho);
double Qnnd1(double x, int N, double D);
double Qsdd1(double x, double *D, long N);
```

DESCRIPTION

delim \$\$ These functions return the virtual waiting time distribution for different queuing models. Parameter x is the amount of unfinished work in the system.

Qmd1() is a model for the M/D/1 queuing system with Poisson arrivals and deterministic (constant) service time. ρ is the load level of the system.

Qnnd1() is the N*D/D/1 queuing system which has constant service time and N deterministic sources with the same period D , so that the load level of system is N/D .

Qsdd1() is the $\sum D_{sub \{i\}} / D/1$ queuing model for a system with number of deterministic sources N , each having its own period, and a constant service time. Table of periods is given by D .

ALGORITHM

M/D/1 waiting time distribution is calculated using three different algorithms:

When $\rho < 0.3$ and $x < (9 + 15 * \text{Log10}(0.3 / \rho))$ the upper limit formula:

$$\sum_{n=x}^{\infty} \frac{(\rho (n-x))^{n-1}}{n!} e^{-\rho (n-x)} (1 - \rho)$$

is used. Terms are calculated logarithmically to avoid overflow.

If $\rho < 0.3$ and $x > (9 + 15 * \text{Log10}(0.3 / \rho))$ or $\rho > 0.3$ and $x > 8$, $Q(x)$ is approximated by

$$\{C_{sub 0}\} \{e^{\sup \{r_{sub 0}\} x}\}, \text{ where}$$

$$\{C_{sub 0}\} = \{1 - \rho\} \text{ over } \{\rho \{e^{\sup \{r_{sub 0}\}} - 1\}\} \text{ and } \{r_{sub 0}\} \text{ is solved from}$$

$$\rho \{e^{\sup \{r_{sub 0}\}} - 1\} - \{r_{sub 0}\} = 0$$

Otherwise if $\rho > .3$ and $x < 8$, Q is calculated with the upper limit sum using an improved algorithm.

N*D/D/1 waiting time distribution is calculated using the following formula:

$$\{Q_{sub X} \sup N\} (x) = \sum_{\{x < n \leq N\}} \left(\{pile \{N \text{ above } n\}\} \right) \sim \left(\{n-x\} \text{ over } D \text{ right} \right) \sup n \sim \left(1 - \{n-x\} \text{ over } D \text{ right} \right) \sup \{N-n\} \sim \{D - N + x\} \text{ over } \{D - n + x\}$$

Since the binomials in the formula would get very large, calculation is done by adding the logarithms of each term. These logarithms can be easily derived from previous terms.

$\sum D_{sub \{i\}} / D/1$ waiting time distribution is given by formula

$$\{Q(x)\} \sim \sum_{\{n>x\}} \left\{ \psi(z_{sub n}) \right\} \text{ over } \{z_{sub n} \sup n-d\} \sim \frac{1}{\sqrt{2\pi}} \frac{\sigma(z_{sub n})}{\left(1 - \sum_{j=1}^N \{\rho_{sub j}\} \text{ over } \{1 - p_{sub j} + p_{sub j} z_{sub n}\} \right)}$$

Values of $z_{sub n}$ are determined from

$$\sum_i \{p_{sub i} z_{sub n}\} \text{ over } \{1 - p_{sub i} + p_{sub i} z_{sub n}\} = n - d$$

An approximating function is used to find the value of z .

ERRORS

When ρ is close to 1, **Qmd1()** might give inaccurate results.

SEE ALSO

COST 224: Performance evaluation and design of multiservice networks