

NAME

Qmd1(), Qnnd1(), Qsdd1() – Virtual waiting time distribution functions

SYNOPSIS

```
#include <queue.h>
```

```
double Qmd1(double x, double rho);
```

```
double Qnnd1(double x, int N, double D);
```

```
double Qsdd1(double x, double *D, long N);
```

DESCRIPTION

delim \$\$ These functions return the virtual waiting time distribution for different queuing models. Parameter x is the amount of unfinished work in the system.

Qmd1() is a model for the M/D/1 queuing system with Poisson arrivals and deterministic (constant) service time. ρ is the load level of the system.

Qnnd1() is the N*D/D/1 queuing system which has constant service time and N deterministic sources with the same period D , so that the load level of system is N/D .

Qsdd1() is the $\sum D_i / D$ queuing model for a system with number of deterministic sources N , each having its own period, and a constant service time. Table of periods is given by D .

ALGORITHM

M/D/1 waiting time distribution is calculated using three different algorithms:

When $\rho < 0.3$ and $x < (9 + 15 * \text{Log}_{10}(0.3 / \rho))$ the upper limit formula:

$$\sum_{n>x} \{ \{ (\rho (n-x)) \sup \{n\} \text{ over } n! \} e^{\sup\{-\rho (n-x)\}} (1 - \rho) \} \}$$

is used. Terms are calculated logarithmically to avoid overflow.

If $\rho < 0.3$ and $x > (9 + 15 * \text{Log}_{10}(0.3 / \rho))$ or $\rho > 0.3$ and $x > 8$, $Q(x)$ is approximated by

$$\{ C_0 \} \{ e^{\sup\{-r_0\} x} \}, \text{ where}$$

$$\{ C_0 \} = \{ 1 - \rho \} \text{ over } \{ \rho \{ e^{\sup\{r_0\}} - 1 \} \} \text{ and } \{ r_0 \} \text{ is solved from}$$

$$\rho (\{ e^{\sup\{r_0\}} - 1 \} - \{ r_0 \}) = 0$$

Otherwise if $\rho > .3$ and $x < 8$, Q is calculated with the upper limit sum using an improved algorithm.

N*D/D/1 waiting time distribution is calculated using the following formula:

$$\{ Q(x) \} = \sum_{x < n \leq N} \left(\{ \text{pile } \{N \text{ above } n\} \text{ right} \} \sim \left(\{ n-x \} \text{ over } D \text{ right} \right) \sup n \sim \left(1 - \{ n-x \} \text{ over } D \text{ right} \right) \sup \{ N-n \} \sim \{ D - N + x \} \text{ over } \{ D - n + x \} \right)$$

Since the binomials in the formula would get very large, calculation is done by adding the logarithms of each term. These logarithms can be easily derived from previous terms.

$\sum D_i / D$ waiting time distribution is given by formula

$$\{ Q(x) \} \sim \sum_{n>x} \{ \{ \psi(z_n) \} \text{ over } \{ z_n \sup n-d \} \sim 1 \text{ over } \{ \sqrt{2 \pi} \} \sigma(z_n) \} \sim \left(1 - \sum_{j=1}^N \{ \rho_j \} \text{ over } \{ 1 - p_j + p_j z_n \} \right)$$

Values of z_n are determined from

$$\sum_i \{ p_i z_n \} \text{ over } \{ 1 - p_i + p_i z_n \} = n - d$$

An approximating function is used to find the value of z .

ERRORS

When ρ is close to 1, **Qmd1()** might give inaccurate results.

SEE ALSO

COST 224: Performance evaluation and design of multiservice networks