

**NAME**

Qmd1(), Qndd1(), Qsdd1() – Virtual waiting time distribution functions

**SYNOPSIS**

```
#include <queue.h>
```

```
double Qmd1(double x, double rho);
```

```
double Qndd1(double x, int N, double D);
```

```
double Qsdd1(double x, double *D, long N);
```

**DESCRIPTION**

delim \$\$ These functions return the virtual waiting time distribution for different queuing models. Parameter  $x$  is the amount of unfinished work in the system.

**Qmd1()** is a model for the M/D/1 queuing system with Poisson arrivals and deterministic (constant) service time.  $\rho$  is the load level of the system.

**Qndd1()** is the N\*D/D/1 queuing system which has constant service time and  $N$  deterministic sources with the same period  $D$ , so that the load level of system is  $N/D$ .

**Qsdd1()** is the  $\sum D_i / D$  queuing model for a system with number of deterministic sources  $N$ , each having its own period, and a constant service time. Table of periods is given by  $D$ .

**ALGORITHM**

M/D/1 waiting time distribution is calculated using three different algorithms:

When  $\rho < 0.3$  and  $x < (9 + 15 * \text{Log}_{10}(0.3 / \rho))$  the upper limit formula:

$$\sum_{n>x} \{ \{ (\rho (n-x)) \sup \{n\} \text{ over } n! \} e^{\sup\{-\rho (n-x)\}} (1 - \rho) \}$$

is used. Terms are calculated logarithmically to avoid overflow.

If  $\rho < 0.3$  and  $x > (9 + 15 * \text{Log}_{10}(0.3 / \rho))$  or  $\rho > 0.3$  and  $x > 8$ ,  $Q(x)$  is approximated by

$$\{ C_0 \} \{ e^{\sup\{-r_0\} x} \}, \text{ where}$$

$$\{ C_0 \} = \{ 1 - \rho \} \text{ over } \{ \rho \{ e^{\sup\{r_0\}} - 1 \} \} \text{ and } \{ r_0 \} \text{ is solved from}$$

$$\rho ( \{ e^{\sup\{r_0\}} - 1 \} - \{ r_0 \} ) = 0$$

Otherwise if  $\rho > .3$  and  $x < 8$ ,  $Q$  is calculated with the upper limit sum using an improved algorithm.

N\*D/D/1 waiting time distribution is calculated using the following formula:

$$\{ Q(x) \} = \sum_{x < n \leq N} \left( \{ \text{pile } \{N \text{ above } n\} \text{ right} \} \sim \left( \{ n-x \} \text{ over } D \text{ right} \} \sup n \sim \left( 1 - \{ n-x \} \text{ over } D \text{ right} \} \sup \{ N-n \} \sim \{ D - N + x \} \text{ over } \{ D - n + x \} \right) \right)$$

Since the binomials in the formula would get very large, calculation is done by adding the logarithms of each term. These logarithms can be easily derived from previous terms.

$\sum D_i / D$  waiting time distribution is given by formula

$$\{ Q(x) \} \sim \sum_{n>x} \{ \{ \psi(z_n) \} \text{ over } \{ z_n \sup n-d \} \sim 1 \text{ over } \{ \sqrt{2 \pi} \} \sigma(z_n) \} \sim \left( 1 - \sum_{j=1}^N \{ \rho_j \} \text{ over } \{ 1 - p_j + p_j z_n \} \right)$$

Values of  $z_n$  are determined from

$$\sum_i \{ p_i z_n \} \text{ over } \{ 1 - p_i + p_i z_n \} = n - d$$

An approximating function is used to find the value of  $z$ .

**ERRORS**

When  $\rho$  is close to 1, **Qmd1()** might give inaccurate results.

**SEE ALSO**

COST 224: Performance evaluation and design of multiservice networks