

NAME

Fmdn() – Virtual waiting time distribution function

IntFmdn() – Virtual waiting time distribution function for integral values of x

Mdn() – Calculates the state probabilities of a queue (queue length)

Fekdn() – Virtual waiting time distribution function

SYNOPSIS

```
#include <queuel.h>
```

```
double Fmdn(double x, double rho, int n);
```

```
double IntFmdn(int N, double rho, int n);
```

```
double *Mdn(double x, double rho, int n);
```

```
double Fekdn(double x, double rho, int k, int n);
```

DESCRIPTION

delim \$\$ These functions return the state probabilities or the virtual waiting time distribution of a $M/D/n$ queue. (And the virtual waiting time distribution of a $E_k/D/n$ queue.)

Fmdn() is a model for the $M/D/n$ queuing system with Poisson arrivals and deterministic (constant) service time. Parameter x is the amount of unfinished work in the system. Rho is the load level of the system and n is the number of servers.

Fekdn() is a model for the $E_k/D/n$ queuing system with Erlang- k arrivals and deterministic (constant) service time. Parameter x is the amount of unfinished work in the system. Rho is the load level of the system and n is the number of servers.

ALGORITHM

$M/D/n$ waiting time distribution is calculated using the following algorithm (Iversen):

$$P\{W \leq t\} = \sum_{i=0}^{n-1} \sum_{j=0}^i P(j) \sum_{\nu=0}^{\lfloor A(t) - i \rfloor} \frac{A(t)^\nu e^{-A(t)}}{\nu!} \frac{1}{1 - \lambda h} \left(\frac{\lambda h}{1 - \lambda h} \right)^i$$

where $P(j)$ is a state probability.

For integral values of the waiting time we have

$$P\{W \leq t\} = \sum_{\nu=0}^{\lfloor n(t+1) - 1 \rfloor} P(\nu)$$

The state probabilities are calculated using the following procedure

– first we make an initial guess $(M/M/n)$:

$$P(0) = \frac{(\lambda h)^n}{n!} \frac{1}{1 - \lambda h} \left(\frac{\lambda h}{1 - \lambda h} \right)^n$$

$$P(i) = P(i-1) \frac{\lambda h}{1 - \lambda h}, \quad i=1, 2, \dots, n-1$$

$$P(i) = P(i-1) \frac{\lambda h}{n}, \quad i=n, n+1, \dots, I \text{ (until } P(i) < \epsilon \text{)}$$

$$P(i) = P(i-1) \frac{\lambda h}{n}, \quad i=I+1, \dots, I+n$$

– then we iterate until $\max_{i \leq I} |P^{(k)}(i) - P^{(k-1)}(i)| < \epsilon$:

$$P^{(k)}(i) = \sum_{\nu=0}^{\lfloor A(t) - i \rfloor} P^{(k-1)}(\nu) \frac{A(t)^\nu e^{-A(t)}}{\nu!} \frac{1}{1 - \lambda h} \left(\frac{\lambda h}{1 - \lambda h} \right)^i + \sum_{\nu=n+1}^{\lfloor A(t) - i \rfloor} P^{(k-1)}(\nu) \frac{A(t)^\nu e^{-A(t)}}{\nu!} \frac{1}{n}$$

$$P^{(k)}(i) = P^{(k)}(i-1) \frac{\lambda h}{n}, \quad i=I+1, \dots, I+n$$

$$S^{(k)} = \sum_{i=0}^{\lfloor A(t) - 1 \rfloor} P^{(k)}(i)$$

$$P^{(k)}(i) = \frac{P^{(k)}(i)}{S^{(k)}}, \quad i=0, 1, \dots, I+n$$

$$P(i, h) = \frac{(\lambda h)^i}{i!} e^{-\lambda h}$$

$E_k/D/n$ waiting time distribution is calculated using the $M/D/n$ algorithm:

$E_k/D/r$ (FIFO) is equivalent to $M/D/r*k$ (FIFO).

ERRORS

When ρ is close to 1, these functions might give inaccurate results.

SEE ALSO

COST 224: Performance evaluation and design of multiservice networks