

NAME

Fend1() – Virtual waiting time distribution function

SYNOPSIS in a C-program

```
#include <queue.h>
```

```
double Fend1(double x, int n, double rho);
```

SYNOPSIS in Mathematica

```
Fend1[Real, Integer, Real]
```

SYNOPSIS with MathLink

```
LnkFend1[Real, Integer, Real]
```

DESCRIPTION

delim \$\$ **Fend1()** is a model for the $\{E \text{ sub } n\}/D/1$ queuing system with Erlang- n arrivals and deterministic (constant) service time. Parameter x is the amount of unfinished work in the system and Rho is the load level of the system.

ALGORITHM

$\{E \text{ sub } n\}/D/1$ waiting time distribution is calculated using the following algorithm:

$$P\{W \leq x\} = n \{F \text{ sub } n\}(x) = n \{e \text{ sup } \{\{\lambda\}x\}\} \{P \text{ sub } mn+n-1\}(x-m).$$

To calculate this probability, we need to resolve the coefficients of polynomials $\{P \text{ sub } k\}$ up to $k = nm+n-1$, where m is the integral part of x .

$$\{P \text{ sub } mn\} = (\{\text{sum}\} \text{ from } \{j=0\} \text{ to } \{mn-n\} \{a \text{ sub } j \text{ sup } (mn-n)\}, \{-\beta\} \text{ over } 1\} \{a \text{ sub } 0 \text{ sup } (mn-1)\}, \{-\beta\} \text{ over } 2\} \{a \text{ sub } 1 \text{ sup } (mn-1)\}, \dots, \{-\beta\} \text{ over } mn\} \{a \text{ sub } mn-1 \text{ sup } (mn-1)\}),$$

$$\{P \text{ sub } mn+i\} = (\{\text{sum}\} \text{ from } \{j=0\} \text{ to } \{mn+i-n\} \{a \text{ sub } j \text{ sup } (mn+i-n)\}, \{-\lambda\} \text{ over } 1\} \{a \text{ sub } 0 \text{ sup } (mn+i-1)\}, \{-\lambda\} \text{ over } 2\} \{a \text{ sub } 1 \text{ sup } (mn+i-1)\}, \dots, \{-\lambda\} \text{ over } mn+1\} \{a \text{ sub } mn+i-1 \text{ sup } (mn+i-1)\}),$$

where $i=1, \dots, n-1$ and $m=1, 2, \dots$

The recursion starts from the initial values $\{a \text{ sub } 0 \text{ sup } (i-1)\} = \{P \text{ sub } i \text{ sup } o\}$:

$$\{P \text{ sub } 0\} = (\{P \text{ sub } 1 \text{ sup } o\}),$$

$$\{P \text{ sub } i\} = (\{P \text{ sub } i+1 \text{ sup } o\}, \{-\lambda\} \text{ over } 1\} \{a \text{ sub } 0 \text{ sup } (i-1)\}, \{-\lambda\} \text{ over } 2\} \{a \text{ sub } 1 \text{ sup } (i-1)\}, \dots, \{-\lambda\} \text{ over } 3\} \{a \text{ sub } i-1 \text{ sup } (i-1)\}), \quad i=1, \dots, n-1.$$

What remains is to determine the initial values $\{P \text{ sub } i \text{ sup } o\}$.

ERRORS

When ρ is close to 1, these functions might give inaccurate results.

SEE ALSO

S. Aalto & J. Virtamo: *M/D/n queue revisited*.