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NAME

Berl_i, Berl_d, Xerl, Aerl – Erlang and inverse Erlang functions

SYNOPSIS

```
#include <erlang.h>
```

```
double Berl_i(int n, double a);
```

```
double Berl_d(double x, double a);
```

```
double Xerl(double b, double a);
```

```
double Aerl(double x, double b);
```

DESCRIPTION

These functions give solutions to the Erlang blocking formula. Erlang formula gives the probability that a communications system with n (or x) lines can't handle any more calls. This also depends on the traffic intensity a . All arguments to these functions must be positive and $0 < b < 1$.

Berl_i() returns the Erlang loss probability for a integer valued trunk group n and Poisson traffic a

Berl_d() returns the Erlang loss probability for a trunk group x , where x is a real valued variable (double). Second parameter is the Poisson traffic a .

Xerl() returns the number of trunks for a loss probability b and Poisson traffic a .

Aerl() returns the Poisson traffic value for a trunk group x and loss probability b .

ALGORITHM

Berl_i() is implemented by using the recursion formula

$$(1) \ E(x+1, a) = \{a E(x, a)\} \text{ over } \{x + 1 + a E(x, a)\}.$$

Berl_d() is divided into five sections depending on the value of the parameters.

Range 1: $x < a$ and $a > 0.01$

Apply the continued fraction expansion formula.

Range 2: $x \geq a$ and $x > 60$

Determine a value x' such that $a > x'$ (i.e. apply an integral valued shift of x). Call **Berl_d** again with $x' < a$ and use the recurrence relation to arrive at the original value of x .

Range 3: $x \geq a$ and $1 \leq x \leq 60$

Apply Rapp's parabola followed by the recurrence relation (1).

Range 4: $x \geq a$ and $a > 0.01$ and $x < 1$

Apply Laguerre quadrature, Abramowitz [2], p. 923

Range 5: $0 < x < 1$ and $0 < a \leq 0.01$

Apply formula (48) from Farmer and Kaufman [ref 1], p. 161

In **Xerl()** we first determine with inequality

$$E(n, a) > E(n+1, a)$$

the position of the interval $[N, N+1]$ such that $N \leq X \leq N+1$. By using the secant iteration method, we determine the exact root of the function

$$f(x) = \text{Berl}_d(x, a) - b.$$

Evaluation time of **Xerl()** is proportional to the result value.

Aerl(): First we compute a starting value for a . Then a is improved using the Newton's iteration scheme. The iteration process is stopped as soon as the relative error in $E(x, a)$ is smaller than some prescribed value. When b is very small, this function has long evaluation time.

REFERENCES

- [1] Farmer, R.F, Kaufman, I., "On the Numerical Evaluation of Some Basic Traffic Formulae", Networks, Vol. 8 (1978) p. 153 - 186
- [2] Abramowitz, M., Stegun, I., "Handbook of mathematical functions", Dover Publications, Inc., New York, 8th printing, 1972.