NAME

Qmd1(), Qndd1(), Qsdd1() − Virtual waiting time distribution functions

SYNOPSIS

#include <queuel.h>

double Qmd1(double x, double rho);

double Qndd1(double x, int N, double D);

double Qsdd1(double x, double *D, long N);

DESCRIPTION

delim $$ These functions return the virtual waiting time distribution for different queuing models. Parameter *x* is the amount of unfinished work in the system.

Qmd1() is a model for the M/D/1 queuing system with Poisson arrivals and deterministic (constant) service time. *Rho* is the load level of the system.

Qndd1() is the N*D/D/1 queuing system which has constant service time and *N* deterministic sources with the same period *D*, so that the load level of system is N/D.

Qsdd1() is the $sum D sub { i } /D/1$ queuing model for a system with number of deterministic sources *N*, each having it's own period, and a constant service time. Table of periods is given by *D*.

ALGORITHM

M/D/1 waiting time distribution is calculated using three different algorithms:

When rho < 0.3 and x < (9 + 15 * Log10 (0.3 / rho) the upper limit formula:
    $sum from {n>x} {{{(rho (n-x)) sup {n}} over n!} e sup{- rho (n-x)} (1- rho)}$
is used. Terms are calculated logarithmically to avoid overflow.

If rho < 0.3 and x > (9 + 15 * Log10(0.3 / rho) or rho > 0.3 and x > 8, Q(x) is approximated by
    $ {C sub 0} {e sup {-{r sub 0} x}}$, where

    ${C sub 0 } = {1 - rho} over {rho {e sup {r sub 0}} - 1}$ and ${r sub 0 }$ is solved from

    $rho ({ e sup {r sub 0}} - 1) - {r sub 0} = 0 $

Otherwise if rho > .3 and x < 8, Q is calculated with the upper limit sum using an improved alogrithm.

N*D/D/1 waiting time distribution is calculated using the following formula:
    ${Q sub X sup N} (x) = sum from {x < n <= N} left ({pile {N above n}} right) ˜ left ({n-x} over D right) sup n ˜ left (1 - {n-x} over D right) sup {N-n} ˜ {D - N + x} over {D - n +x} $
Since the binomials in the formula would get very large, calculation is done by adding the logarithms of each term. These logarithms can be easily derived from previous terms.

$SIGMA D sub {i}/D/1$ waiting time distribution is given by formula
    $Q(x) ˜=˜ sum from {n>x} {{ psi (z sub n)} over {z sub n sup n-d} ˜ 1 over {sqrt {2 pi} sigma (z sub n) } ˜ left (1 - sum from {j=1} to N {{rho sub j} over {1 - p sub j + p sub j z sub n}} right)} $
Values of $z sub n$ are determined from
    $sum from i {p sub i z sub n} over {1 - p sub i + p sub i z sub n} = n - d$
An approximating function is used to find the value of z.

ERRORS

When $ rho $ is close to 1, **Qmd1()** might give inaccurate results.

SEE ALSO

COST 224: Performance evaluation and design of multiservice networks