

1. A machine is taken out of production if it fails, or after 5 hours, whichever comes first. It has been found empirically that the time to failure obeys the exponential distribution with mean value 2 hours. Give an algorithm for generating samples of the time until the machine is taken out of production.
2. The variate X has a distribution with the pdf (plot the density)

$$f(x) = \begin{cases} \frac{1}{2}(x-2), & 2 \leq x \leq 3 \\ \frac{1}{2}(2 - \frac{x}{3}), & 3 < x \leq 6. \end{cases}$$

Give an inverse transformation algorithm for for generating X from this distribution.

3. Give an acceptance-rejection algorithm for the previous problem.
4. Devise an algorithm by using the composite method to generate rv:s obeying the density

$$f(x) = \frac{1}{2} + x^3 + 2x^7, \quad 0 \leq x \leq 1.$$

Hint: compute the cumulative distribution.

5. Give an acceptance-rejection algorithm for generating a variate X from the pdf

$$f(x) = x e^{-x}, \quad x \geq 0$$

using the auxiliary pdf $g(x) = 0.3 e^{-0.3x}$. Hint: determine $c \cdot g(x)$ so that it touches $f(x)$ at one point.

What is the acceptance ratio with these parameters, i.e., the probability that a sample is accepted?

Implement the rejection method (by using, e.g., Matlab, Mathematica, or Excel). Make 1000 trials with the algorithm, and calculate the average value of X from the accepted trials. Verify that the samples are generated correctly (i.e., that the mean value of your accepted samples and the acceptance ratio are correct).