1. Empty taxis pass a street corner at a Poisson rate of 2 per minute and pick a passenger if one is waiting there. Passengers arrive at the street corner at rate 1 per minute and wait for a taxi only if there are fewer than four persons waiting; otherwise they leave and never return. Find the average waiting time of a passenger who joins the queue.

2. Consider an $M/M/1$ queue with the following variation. Whenever a service is completed a departure occurs only with probability α. With probability $1-\alpha$ the customer, instead of leaving, joins the end of queue.

   a) Draw the state transition diagram and solve the steady state probabilities.
   b) Find the expected waiting time of a customer from the time he arrives until he enters service for the first time.
   c) What is the probability that a customer enters service exactly n times?
   d) What is the expected amount of time that a customer spends in service (not including the time spent waiting in line).

3. Show that for Erlang's C-function (probability that an arriving customer have to wait in $M/M/n$ system) it holds that

$$\frac{1}{C(n,a)} = \rho + \frac{1-\rho}{E(n,a)}$$

   where n is the number of servers, $a = \lambda/\mu$, $\rho = a/n$ (load per server) and $E(n,a)$ is Erlang's B-function.

4. $M/M/2$ system with heterogeneous servers. Derive the stationary distribution of an modified $M/M/2$ system where two servers have different service rates, μ_1 and μ_2. A customer that arrives when the system is empty is routed to the server 1. Draw the state transition diagram and deduce the steady state distribution. Hint: the state where there is only one customer in the system must be split into two states depending on in which server the customer is.

5. Customers arrive at an $M/G/1$ system according to a Poisson process with rate λ. The service of each customer comprises of k different steps (only after all these steps have been accomplished can the next customer be taken into service). Each of the steps takes independently an exponentially distributed time, Exp(μ). Find the mean waiting and sojourn times of a customer in the system?

6. Consider a simplified model for TCP link.[1] Assume that TCP packets arrive according to a Poisson process with arrival intensity of $\lambda = 100$ pkt/s to a 2 Mbit/s DSL-modem acting as a router. The packet length distribution and respective service times are the following:

length	proportion	time / ms
40	0.1	0.16
576	0.3	2.3
1500	0.6	5.9

   Determine the mean waiting time of a packet in the queue, when the service discipline is,

   a) FIFO
   b) the shortest job first (non-preemptive)

[1] Typically there are three peaks in TCP packet length distribution: the first peak at 40 bytes (ACK), the second peak around 552/576 bytes (the smallest possible value for MTU) and the third at 1500 bytes (the largest possible IP packet in ethernet).