HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory, II/2006

Exercise 3
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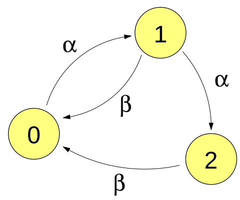
1. Trials are performed in sequence. If the last two trials were successes, then the next trial is success with probability 0.8; otherwise the next trial is a success with probability 0.5. In the long run, what proportion of trials are successes? Instruction: Study the system as a 4-state Markov chain, where states of the system are defined by the results of the last two trials.

2. A continuous-time Markov process has the state transition diagram depicted in the figure, where $\alpha = 1$/s, $\beta = 2$/s.



   a) Determine the probability distribution π_i in equilibrium.
   b) Determine the probability distribution $\pi_i^{(e)}$ in equilibrium in the respective embedded Markov chain.
   c) What are the average lifetimes $\bar{T}_i$ of different states? Verify numerically that $\pi_i = \pi_i^{(e)}\bar{T}_i / \sum_j \pi_j^{(e)}\bar{T}_j$.

3. Determine the probability distribution in equilibrium for birth-death processes (state space $i = 0, 1, 2, \ldots$), which transition intensities are a) $\lambda_i = \lambda$, $\mu_i = i\mu$, b) $\lambda_i = \lambda/(i+1)$, $\mu_i = \mu$, where λ and μ are constants.

4. A link of a packet network carries on the average 10 packets per second. The packets can be assumed to arrive according to a Poisson process. Each packet is an acknowledgment (ACK) packet with the probability 30 %, independent of the others. Consider an arbitrary interval of length of one second.

   a) What is the probability that at least one ACK packet has been on the link?
   b) What is the expected number of all packets given that 5 ACK packets have been observed on the link?
   c) Given that 8 packets have been observed in total, what is the probability that two of them are ACK packets?

5. In a traffic measurement the number of incoming calls N is observed daily from 3:00pm to 3:10pm. It can be assumed that the arrivals obey the Poisson process with intensity λ, which is constant during the given time period, but varies from day to day. According to the measurements the expected value of N is 25 and its variance is 29. What are the expected value and the standard deviation of λ? Hint: apply the conditioning rules for mean and variance: $\mathrm{E}[X] = \mathrm{E}[\mathrm{E}[X|Y]]$ and $\mathrm{V}[X] = \mathrm{E}[\mathrm{V}[X|Y]] + \mathrm{V}[\mathrm{E}[X|Y]]$.

6. Cars pass point A in a highway with an average interval of 10 minutes. The intervals are independent and identically distributed with standard deviation (i.e. the square root of variance) of 6 minutes. A hitch-hiker arrives (randomly) at point A. What is the mean waiting time until the next car arrives?