HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.3143 Queueing Theory, II/2006

Exercise 1
7.11.2006
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1. A connection consists of 4 unreliable consecutive links. On each link the probability that a transmitted bit (0 or 1) is received correctly is 90% and with probability of 10% the received bit has flipped into the other one. What is the probability that a transmitted bit is received correctly at the other end of the connection?

2. In each n boxes there are $m > n$ balls. In box $i,\ i = 1, \ldots, n,$ there are i red balls and $m - i$ green balls. One of the boxes is chosen randomly and one ball is picked up from the box. It turns out that the ball is red. What is the probability that the chosen box was box j?

3. Assume that continuous random variables X and Y have a joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 2 - x - y, & \text{when } 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   a) Determine the marginal distributions $f_X(x)$ and $f_Y(y)$. Are X and Y independent?

   b) Determine the conditional density functions $f_{X|Y}(x,y)$ and $f_{Y|X}(y,x)$.

   c) Calculate $\mathrm{E}[X|Y = y]$ and $\mathrm{E}[Y|X = x]$.

4. Apply the conditioning rules

$$\begin{aligned} \mathrm{E}[X] &= \mathrm{E}[\mathrm{E}[X|Y]] \\ \mathrm{V}[X] &= \mathrm{E}[\mathrm{V}[X|Y]] + \mathrm{V}[\mathrm{E}[X|Y]] \end{aligned}$$

   to the case $X = X_1 + \ldots + X_N$, where the X_i are independent and identically distributed (i.i.d.) random variables with mean m and variance σ^2, and N is a positive integer valued random variable with mean n and variance ν^2. Hint: Condition on the value of N.

5. Let K_1 and K_2 be two non-negative integer valued random variables with point probabilities $\mathrm{P}\{K_1 = k\} = (1-\alpha)\alpha^k$ and $\mathrm{P}\{K_2 = k\} = (1-\beta)\beta^k$, $k = 0, 1, \ldots$. Determine the distribution of sum $K = K_1 + K_2$, $\mathrm{P}\{K = k\}$, $k = 0, 1, \ldots$. Hint: use generating functions and partial fraction decomposition.

6. Let X be an integer valued random variable, $X = 0, 1, 2..$, and $\mathcal{G}(z)$ its generating function. It is known that by using the generating functions $\mathrm{E}[X] = \frac{d}{dz}\mathcal{G}(z)|_{z=1} = \mathcal{G}'(1)$ and $\mathrm{E}[X^2] = \frac{d}{dz}z\frac{d}{dz}\mathcal{G}(z)|_{z=1} = \mathcal{G}''(1) + \mathcal{G}'(1)$. Assume, that X is positive, that is $\mathrm{P}\{X = 0\} = 0$. Express a) $\mathrm{E}[1/X]$ and b) $\mathrm{E}[1/X^2]$ in terms of the generating function. Hint: integrate.