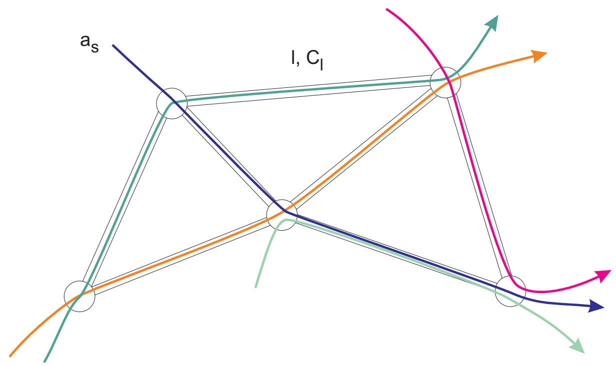
CALCULATING BLOCKING IN A NETWORK

Erlang's function $E(C, a)$ allows us to calculate the blocking probability in a single link with capacity C (trunks) and offered a traffic with intensity a.

In the following we investigate how the blocking probability can be approximately calculated in a real circuit switched network which consists of several links carrying several traffic streams.

$$\begin{cases} s & = \text{traffic stream} \\ \ell & = \text{link, } C_\ell \text{ trunks} \\ R_s & = \text{route of stream } s \text{ (sequence of links)} \\ a_s & = \text{offered traffic intensity of stream } s \\ A_\ell & = \text{traffic intensity offered to link } \ell \\ B_s & = \text{the blocking of stream } s \\ b_\ell & = \text{blocking probability in link } \ell \end{cases}$$



Denote further $\ell \in R_s$ when link ℓ is on the route of stream s.

The task is to calculate the blocking probabilities B_s for streams s.

If the blocking probabilities are small, then approximately $$\boxed{B_s = \sum_{\ell \in R_s} b_\ell}$$

If the link blockings are independent, then $$\boxed{1 - B_s = \prod_{\ell \in R_s} (1 - b_\ell)}$$

The problem is how to calculate the link blockings b_ℓ.

Reduced load approximation

This is also known as Erlang's fixed point method.

When the blocking is small the intensity of the traffic offered to link ℓ is approximately

$$A_\ell = \underbrace{\sum_{s:\,\ell\in R_s} a_s}_{\text{sum over the streams using link } \ell}, \qquad b_\ell = E(C_\ell, A_\ell)$$

If the blockings are not small one has to account for the traffic thinning in other links.

$$\begin{cases} a_s^\ell & \text{the thinned traffic offered by stream } s \text{ to link } \ell \\ A_\ell = \sum_{s:\,\ell\in R_s} a_s^\ell & \text{sum of the thinned traffic streams offered to link } \ell \end{cases}$$

$$a_s^\ell = \begin{cases} 0, & \text{if } \ell \notin R_s \\ a_s \cdot \underbrace{\prod_{k\in R_s-\{\ell\}} (1-b_k)}_{\text{thinning factor in other links}} = a_s \cdot \dfrac{1}{1-b_\ell} \prod_{k\in R_s} (1-b_k), & \text{if } \ell \in R_s \end{cases}$$

Reduced load approximation (continued)

The blocking probability of link ℓ can now be written as

$$\boxed{b_\ell = E\Big(C_\ell, \sum_{s:\,\ell\in R_s} a_s \frac{1}{1-b_\ell} \prod_{k\in R_s} (1-b_k)\Big)}$$

The blocking in link ℓ depends on the blockings in other links.

- Set of equations to determined the link blockings
  - one equation for each link
  - the equations depend on each other
  - nonlinear set of equations
- The set of equations can be solved iteratively
  - initial guess: $b_\ell = 0, \ \forall \ell$
  - substitute this into the right hand side
  - new values are obtained for the b_ℓ
  - substitute again into the rhs and continue until nothing is changes

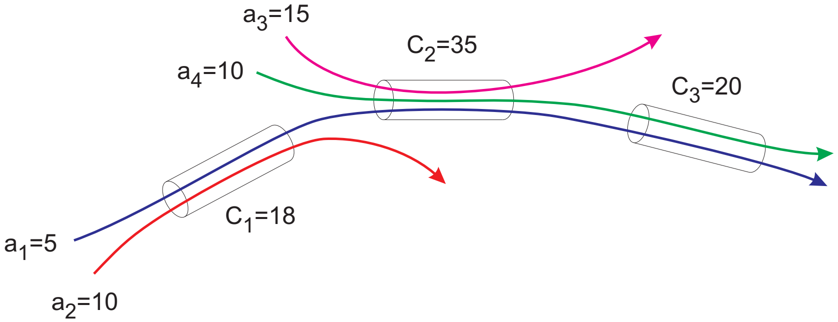
$\Rightarrow$ Erlang's fixed point method (refers to the method of solving the set of equations)

Given the link blocking probabilities b_ℓ, one gets the blocking probability of stream s:

$$\boxed{B_s = 1 - \prod_{\ell\in R_s} (1-b_\ell)}$$

Example

$$\begin{cases} b_1 = E(18,\ 5(1-b_2)(1-b_3)+10) \\ b_2 = E(35,\ 5(1-b_1)(1-b_3)+15+10(1-b_3)) \\ b_3 = E(20,\ 5(1-b_1)(1-b_2)+10(1-b_2)) \end{cases}$$



Initial guess $b_1 = b_2 = b_3 = 0$.

Iteration	b_1	b_2	b_3
1	8.6 %	5.4 %	4.6 %
2	7.5 %	4.2 %	2.7 %
3	7.8 %	4.5 %	3.0 %
4	7.7 %	4.4 %	2.9 %
5	7.7 %	4.5 %	3.0 %
6	7.7 %	4.5 %	3.0 %

Note. One gets alternatingly upper and lower bounds. In the first round, when the unthinned traffics are offered to the link, one overestimates the link blockings. When the streams are thinned by too high blockings, in the next round one gets too small traffics blockings etc. The right value is between two consecutive iterates.

End-to-end blocking probabilities of the traffic streams

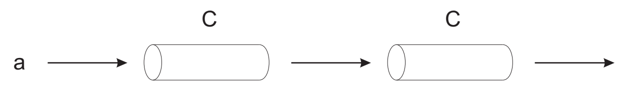
$$\begin{cases} B_1 = 1-(1-b_1)(1-b_2)(1-b_3) = 14.4\,\% \\ B_2 = 1-(1-b_1) = 7.7\,\% \\ B_3 = 1-(1-b_2) = 4.5\,\% \\ B_4 = 1-(1-b_2)(1-b_3) = 7.3\,\% \end{cases}$$

Why is this an approximation?

- The method only considers the thinning of the traffic streams
- In reality, the traffic offered to link ℓ is not Poissonian
  - it is smoother because the blockings in the other links cuts the traffic peaks
- The blockings in different links are not independent.
- Approximation is good (asymptotically exact), if there is big number of streams in each link and none of the streams is dominant.

The worst case from the point of view of the approximation

- Only one stream (links are certainly not independent!)
- Two links with the same capacity (C)
- Actually only the first link blocks
  - all the traffic that passes the first link will pass also the second link



The reduced load approximation, however, calculates the blocking in both of the links (albeit with reduced loads).

$$\begin{cases} b_1 = E(C, a(1-b_2)) \\ b_2 = E(C, a(1-b_1)) \end{cases} \quad \text{By the symmetry, } b_1 = b_2 = b, \quad \begin{cases} b = E(C, a(1-b)) \\ B = 1-(1-b)^2 = 2b - b^2 \end{cases}$$

Example. $a = 10$, $C = 15$

The correct solution: $B = E(15, 10) = 3.7\,\%$

The reduced load method:

$b = E(15, 10(1-b)) \Rightarrow b = 3.1\,\%$

$B = 1 - (1-b)^2 = 6.1\,\%$