HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.3141 Teletraffic Theory, IV/2008

Exercise 5
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1. A link with capacity 20 Mbit/s carries identical connections, each of which has an instantaneous bit rate varying according to the exponential distribution with mean 1 Mbit/s. How many connections of this type can be multiplexed if the acceptable loss probability P_{loss} is 10^{-6}? Use the approximation formula for P_{loss} given in the lecture material.

2. Prove the Kaufman-Roberts recursion formula

$$c\,Q(c) = \sum_i b_i a_i Q(c - b_i)$$

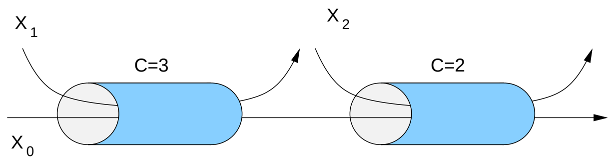
where $Q(c)$ denotes the probability that c capacity units (trunks) are occupied, b_i denotes the bandwidth requirement of class-i calls and $a_i = \lambda_i/\mu_i$ is the offered traffic intensity of class-i calls.

Hint: Consider any state where the capacity occupancy is c. Write the detailed balance equation between this state and the state where there is one less calls of class i. Multiply this equation by b_i and sum over all classes. Finally, sum over all the states where the capacity occupancy is c.

3. Find the exact blocking probabilities for connection types $i = 0, 1, 2$ in the network shown in the figure using the formula

$$B_i = 1 - \frac{G(\mathbf{C} - \mathbf{b}^{(i)})}{G(\mathbf{C})}$$

where $G(\mathbf{C})$ is the sum of the unnormalized state probabilities over the state space limited by the link capacities $\mathbf{C} = (C_1, C_2)$ and $G(\mathbf{C} - \mathbf{b}^{(i)})$ is the corresponding sum over the state space limited by link capacities that are otherwise the same but along the route of class-i connections the capacities are reduced by the capacity required by one class-i connection. All classes $i = 0, 1$ and 2 have the same parameters $(a_i, b_i) = (0.5, 1)$; a class-0 connection requires the same capacity $b_0 = 1$ on both links.



4. Calculate the same approximately by using the reduced load approximation.

5. A TCP connection has an average RTT of 250 ms, and packets are lost along the connection with probability 1.5%. Calculate the average throughput of the connection in packets/s. What is the bit rate if the connection uses maximum packet size of 1500 bytes?

6. The average throughput of a TCP connection can be derived also by another reasoning (than the one presented in the lectures). In the congestion avoidance mode the flow control of TCP realizes a so-called AIMD mechanism (additive increase - multiplicative decrease). Assume the average size of the congestion window is W and the packet loss probability is p. So, each sent packet is lost with probability p, which leads to a decrease of the congestion window by amount $W/2$, and successfully delivered with probability $1-p$, which leads to an increase

of the congestion window by the amount $1/W$ (the congestion window is increased by 1 for each full round trip time, i.e. for W sent packets). In the steady state the average size of the congestion window does not change. What is its value in the equilibrium? Derive an expression for the throughput (differs slightly from the expression given in the lecture material).