HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.3141 Teletraffic Theory, IV/2008

Exercise 4
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1. The output port of an ATM switch carries 4 constant bit rate virtual channel connections. The speed of the link is 155 Mbit/s and the bit rate (net information rate) of each connection is 24 Mbit/s. The information is packed into the 48 octet (byte) length payload of the cell, which additionally has a header of 5 octets. Using the $N * D/D/1$ model calculate the probability that there are at least n cells in the output buffer of the port for the values $n = 0, \ldots, 4$.

2. Cells arrive to a modulated $N * D/D/1$ queue from three different sources. In each of the streams the cell interarrival time is 5 (cell transmission times) when the burst is active. The activity probabilities of the sources are 0.5, 0.4 and 0.2. Calculate the probabilities that there are n cells, $n = 0, \ldots, 3$ in the queue.

3. Let the unfinished work in a queue, X, measured in the time it takes to serve the work (also called the virtual waiting time), have the tail distribution $Q(x) = \mathrm{P}\{X > x\}$. Denote the actual waiting time of random customer by W and its tail distribution by $W(x) = \mathrm{P}\{W > x\}$. Justify the following: a) in an $M/D/1$ queue it holds that $W(x) = Q(x)$, b) in an $N*D/D/1$ queue it holds that $W_N(x) = Q_{N-1}(x)$, where the subscript refers to the number of sources in the system.

4. Customers arrive at an M/D/1 queue with Poissonian rate λ, each customer bringing an amount d of work in the queue. The server has rate C and thus the load of the system is $\rho = \lambda d/C$. The tail distribution of the unfinished work X in the queue is known to be asymptotically of exponential form $G(x) = \mathrm{P}\{X > x\} = Ae^{-kx}$, where A and k are some constants. Derive an equation for k by writing the balance condition of the probability flows across a surface at level x ($x \gg d$). Hint: 1) as the server is discharging the queue, the probability mass with density $-G'(x)$ at point x flows at rate C downwards, 2) every arrival that finds the system in a state X with $x - d < X \leq x$ transfers a probability mass of 1 across the surface. Solve the equation for k when $\rho = 0.5$.

5. Determine the twisted distribution and its mean and variance for a random variable X, which obeys
a) Binomial distribution Bin(N, p),
b) Poisson distribution Poisson(a).

6. The bit rate produced by a traffic source varies as follows: 50 % of the time 0 kbit/s, 30 % of the time 100 kbit/s and 20 % of the time 300 kbit/s. How many sources of this type can be multiplexed on link with capacity 2 Mbit/s, when the allowed loss probability is $P_{\mathrm{loss}} \leq 10^{-4}$? Thus, what is the effective bandwidth of one source in this setting? Compare with the mean and peak rates. Hint: Use the approximation formula at the bottom of page 17 of the lectures.