1. The output port of an ATM switch carries 4 constant bit rate virtual channel connections. The speed of the link is 155 Mbit/s and the bit rate (net information rate) of each connection is 24 Mbit/s. The information is packed into the 48 octet (byte) length payload of the cell, which additionally has a header of 5 octets. Using the \( N \cdot D/D/1 \) model calculate the probability that there are at least \( n \) cells in the output buffer of the port for the values \( n = 0, \ldots, 4 \).

2. Cells arrive to a modulated \( N \cdot D/D/1 \) queue from three different sources. In each of the streams the cell interarrival time is 5 (cell transmission times) when the burst is active. The activity probabilities of the sources are 0.5, 0.4 and 0.2. Calculate the probabilities that there are \( n \) cells, \( n = 0, \ldots, 3 \) in the queue.

3. Let the unfinished work in a queue, \( X \), measured in the time it takes to serve the work (also called the virtual waiting time), have the tail distribution \( Q(x) = P\{X > x\} \). Denote the actual waiting time of random customer by \( W \) and its tail distribution by \( W(x) = P\{W > x\} \). Justify the following: a) in an \( M/D/1 \) queue it holds that \( W(x) = Q(x) \), b) in an \( N \cdot D/D/1 \) queue it holds that \( W_N(x) = Q_{N-1}(x) \), where the subscript refers to the number of sources in the system.

4. Customers arrive at an \( M/D/1 \) queue with Poissonian rate \( \lambda \), each customer bringing an amount \( d \) of work in the queue. The server has rate \( C \) and thus the load of the system is \( \rho = \frac{\lambda d}{C} \). The tail distribution of the unfinished work \( X \) in the queue is known to be asymptotically of exponential form \( G(x) = P\{X > x\} = Ae^{-kx} \), where \( A \) and \( k \) are some constants. Derive an equation for \( k \) by writing the balance condition of the probability flows across a surface at level \( x \) \((x \gg d)\). Hint: 1) as the server is discharging the queue, the probability mass with density \(-G'(x)\) at point \( x \) flows at rate \( C \) downwards, 2) every arrival that finds the system in a state \( X \) with \( x - d < X \leq x \) transfers a probability mass of 1 across the surface. Solve the equation for \( k \) when \( \rho = 0.5 \).

5. Determine the twisted distribution and its mean and variance for a random variable \( X \), which obeys
   a) Binomial distribution \( \text{Bin}(N, p) \),
   b) Poisson distribution \( \text{Poisson}(a) \).

6. The bit rate produced by a traffic source varies as follows: 50% of the time 0 kbit/s, 30% of the time 100 kbit/s and 20% of the time 300 kbit/s. How many sources of this type can be multiplexed on link with capacity 2 Mbit/s, when the allowed loss probability is \( P_{\text{loss}} \leq 10^{-4} \)? Thus, what is the effective bandwidth of one source in this setting? Compare with the mean and peak rates. Hint: Use the approximation formula at the bottom of page 17 of the lectures.