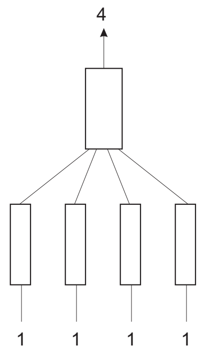
HELSINKI UNIVERSITY OF TECHNOLOGY — **Exercise 3**
Department of Communications and Networking — 7.4.2008
S-38.3141 Teletraffic Theory, IV/2008 — Virtamo / Penttinen

1. A packet network consists of a common link and four access links attached to it. The incoming traffic to each access link is 1 Mbit/s and the specific costs (cost per capacity unit) of all the links are equal. We can afford a total capacity of all the links of 14 Mbit/s. Divide this capacity to different links in an optimal way such that the mean delay of the packets is minimized (using so-called square root method).



2. (a) Determine the optimal capacity allocation (capacities C_i) for the starlike packet network of Fig. 2, when the specific costs of all the links are equal and the total capacity available for all the links together is $C = 45$ kbit/s. The arrival rates λ_i to each node are indicated in the figure. The mean size of the messages is $1/\mu = 1000$ bit. Calculate also the mean delay in each link and the average delay of a randomly chosen packet that arrives to the network.

   (b) Repeat the same for the treelike network of Fig. 3.

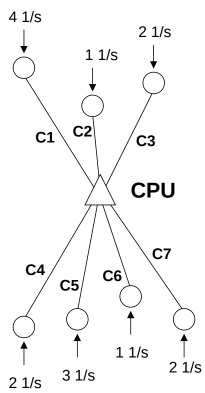


Figure 2.

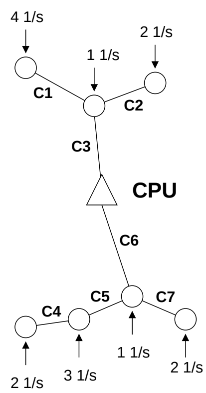


Figure 3.

3. Consider a (hypothetical) circuit-switched telephone network consisting of two consecutive links with capacity $C = 4$. So, on each link, there can be at most 4 ongoing calls simultaneously. New calls arrive according to a Poisson process with rate 20 calls per hour, and the call holding times are independently and exponentially distributed with mean 3 minutes. Each call passes through both the links. Calculate the call blocking probability a) exactly, b) by using the Reduced Load Approximation.

4. In a hexagonal ring network with 6 nodes, the offered traffic between each pair of nodes is 2 erl and the traffic is carried along the shortest route (in the case of two paths of equal length,

the traffic is split evenly between these paths). All the links have the capacity of 16 trunks. Using the reduced-load approximation, calculate the link blocking probability and the end-to-end blocking probabilities for the three different connection types (of different lengths).

5. In an input buffered 2×2 ATM switch the maximum throughput limited by the HOL blocking is 0.75, when the destination addresses of the incoming cells are distributed uniformly among the output ports and are independent for each incoming cell. Here we investigate how the throughput is affected when these assumptions are relaxed:

   a) The destination addresses are uniformly distributed (i.e. the two output ports are equally probable) but the addresses on the incoming cells are correlated in such a way that next cell in a given input has the same destination port as the previous one with the probability 2/3 and different destination port with the probability 1/3.
   b) The destination addresses on different cells are independent but the probabilities of the destination ports are different (so called hot spot traffic): 3/4 and 1/4. Hint: in this case you need three states to describe the state of the cells in the HOL position.

6. Derive the tail probability $Q(x) = \mathrm{P}\{X > x\}$ of an $M/M/1$ queue by applying the Beneš method and using the normal approximation for the amount of work arriving in a given interval. Auxiliary result: the following holds identically $\int_0^\infty e^{-\frac{1}{2}(ax+b/x)^2} dx = \sqrt{\frac{\pi}{2}} e^{-2ab}/a$.