Simulation Studies on Performance of Balanced Fairness

Vesa Timonen

Networking Laboratory

Instructor & supervisor: professor Jorma Virtamo
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Objectives

• Survey on concept of fairness
  – Focus on concept of balanced fairness
• To examine throughputs and sensitivity in different network topologies with three allocation policies: balanced, max-min and proportional
• To verify analytical results
Network model

• Assumptions
  – Packet level observations discarded
  – Fluid model – flows as continuous streams, no storing of data in queues or links
  – Propagation delays discarded – immediate changes, no delays at transfers
  – When a flow starts, it is immediately received at the destination at the sending rate
Network model (cont.)

- **Network model**
  - Set of links with finite fixed capacity
  - Set of fixed unique routes as flow classes
  - Network state is described by vector $\mathbf{x}$ containing number of active flows of each flow class
  - Feasibility: allocated capacity may not exceed network resources
  - Traffic condition: traffic load may not exceed network resources
Fairness

• Classical fairness in static network scenario
• Max-min fairness
  – Traditional definition of fairness
  – All flows get as equal rate as possible
• Proportional fairness
  – “deviation from the fair allocation causes a negative average change”
Fairness (cont.)

• Utility-based fairness – generalization to optimization problem \( \max \sum_{r \in R} u^r_{\alpha}(\phi_r) \), where

\[
u^r_{\alpha}(\phi_r) = \begin{cases} 
w_r x_r \log(\phi_r/x_r), & \alpha = 1 \\
w_r x_r \frac{(\phi_r/x_r)^{1-\alpha}}{1-\alpha}, & \alpha \neq 1 \end{cases}
\]

• For class \( r \)
  \( x_r : \) nof active flows
  \( \phi_r : \) the allocated capacity
  \( w_r : \) the weight

• Parameter \( \alpha \) defines the fairness criterion:
  \( \alpha \to 0 \) Throughput max.
  \( \alpha \to 1 \) Proportional fairness
  \( \alpha \to 2 \) Potential delay min.
  \( \alpha \to \infty \) Max-min fairness
Balanced Fairness

• Balance property

\[
\frac{\phi_k(x-e_k')}{\phi_k(x)} = \frac{\phi_k(x-e_k)}{\phi_k'(x)}, \forall x : x_k > 0
\]

– The experienced relative change in allocation of one flow class caused by the removal of a flow of the other class, is equal for all flow class pairs

• Balance function

– Path from state 0 to state \( x : \langle x, x-e_{k_1}, x-e_{k_1}-e_{k_2}, \ldots, x-e_{k_1}-\ldots-e_{k_{n-1}} \rangle \)

– Balance function \( \Phi(x) = \frac{1}{\phi_{k_1}(x)\phi_{k_2}(x-e_{k_1})\ldots\phi_{k_n}(x-e_{k_1}-\ldots-e_{k_{n-1}})} \).

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Balanced Fairness (cont.)

• Balanced allocation
  – Capacity allocation for flow class \( k \) defined by the balance function is
    \[ \phi_k = \frac{\Phi(x - e_k)}{\Phi(x)}, \forall k : x_k > 0 \]

• Balanced fairness
  – No of different balanced allocations is infinite
  – Balanced fairness is unique allocation defined by recursion
    \[ \Phi(x) = \max_{l \in L} \left\{ \frac{1}{C_{l, i \in r_i}} \sum \Phi(x - e_i) \right\} \]
    – defines the most efficient balanced allocation
Balanced Fairness (cont.)

• Properties
  – Insensitive to
    • Flow size distribution
    • Distribution of the nof flows per session
    • Correlation between successive flow sizes and think-time durations
  – Necessitates that session arrivals are Poisson
  – Distribution of the nof flows in progress and expected throughput depend only on the average load of each flow class
Simulations and Results

• Simulations
  – Three different allocation policies – balanced fairness, max-min fairness and proportional fairness
  – Throughput
    • Homogenous and heterogeneous traffic
  – Sensitivity
    • Unimodal, bimodal and uniform flow size distributions
    • Time-scale variation – constant demand with varying ratio of flow size and arrival rate
  – Flow duration variance
  – Slow-down factor
Simulations and Results (cont.)

• Setups
  – Linear network with 2 and 5 links
  – Parking lot network with 3, 4 and 5 links
  – 2×2 grid network
  – Hypercycle network with 3, 4 and 5 links
  – 4 different tree networks
Simulations and Results (cont.)

- **Results**
  - **Throughput**
    - In general, max-min fairness prefers long routes more than balanced fairness or proportional fairness
    - Differences between balanced fairness and utility-based criteria are quite small
  - **Sensitivity**
    - Balanced fairness is insensitive
    - Also the utility-based criteria are quite insensitive
Simulations and Results (cont.)

• Verified analytical results
  – Proportional fairness coincides with balanced fairness in hypercube network topologies (lines, grids)
  – Utility-based allocation coincide in tree network topologies (parking lot, trees)
  – Simulated throughputs followed exactly the analytical throughput curves in case of balanced fairness

• Main result
  – Simulations showed that balanced fairness provides an effective tool to approximate and evaluate the performance metrics in analytical way
Summary

• This study presented
  – Classical fairness criteria
  – Notion of balanced fairness
  – Main simulation results and verified analytical results

• Questions?