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Master thesis

Performance Evaluation of Serially Concatenated Space-Time Codes

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Outline

- Introduction
- Concatenated codes and iterative decoding
- Space-time codes
- Serially concatenated space-time codes
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The history of channel coding dates back to Shannon’s pioneering work 1948 predicing that reliable communication over a noisy channel is possible by using channel coding.

First practical channel code was the single error correction block code by Hamming 1950.

Convolutional codes were discovered by Elias in 1955.

Viterbi algorithm to decode Convolutional codes was presented in 1967 by Andrew J. Viterbi.

Concatenated codes was initiated by Forny in 1966.

The minimum bit error rate decoding algorithm was presented 1974 by Bahl et al.

Space-time codes were presented by Tarokh 1997. Space-time codes have good bandwidth efficiency and poor coding gain.

Serial concatenation of space-time codes with other channel codes to improve the coding gain of space-time codes
Concatenated codes and iterative decoding 1/7

- Powerful codes can be obtained by concatenating two codes or more in serial or parallel.

- Turbo codes are parallel concatenated Convolutional codes (PCCC) that have performance near to the theoretical Shannon’s capacity limit.

- Serially concatenated Convolutional codes (SCCC) are another configuration of concatenated codes that in some cases have performance superior to turbo codes.

- Iterative decoding of concatenated codes based on soft output algorithms such as
  - Maximum A posteriori Probability (MAP)
  - Soft Output Viterbi Algorithm (SOVA)
**Concatenated codes and iterative decoding 2/7**

- **PCCC Encoder structure**
  - Both encoders are Systematic Convolutional codes.
  - The Encoder 1 has rate k/ n1.
  - The Encoder 2 has rate k/ n2.
  - The Overall PCCC codes rate k/ (n1+n2-k).
  - The interleaver Π is an essential feature of the PCCC
  - The PCCC has encoding latency of N (interleaver size).
  - The PCCC has overall latency of 2N.
  - In delay sensitive applications trade off between performance and delay should be considered.
  - Systematic recursive Convolutional codes (SRCC) have better performance than non recursive.

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**Figure 1** PCCC encoder block diagram
Concatenated codes and iterative decoding 3/7

- PCCC decoder structure
  - The two decoders use a Soft Input Soft Output (SISO) module.
  - The first decoder performs the decoding and outputs soft decision that is interleaved and used by the second decoder.
  - The second decoder performs the decoding and output soft decision that is de-interleaved and used by the first decoder again.
  - The process is iterated many times before the final decision is made.

![PCCC decoder block diagram](image)

**Figure 2** PCCC decoder block diagram
Concatenated codes and iterative decoding 4/7

- **SCCC Encoder structure**
  - Both encoders are Convolutional encoders.
  - The outer encoder has rate $k_o/n_o$.
  - The encoder has rate $k_i/n_i$.
  - The Overall SCCC codes rate $k_i k_o/n_i n_o$.
  - The interleaver $P$ is an essential feature of the SCCC.
  - The SCCC has encoding latency of $N* k_o/n_o$.
  - The SCCC has overall latency of $2* N * k_o/n_o$.
  - In delay sensitive applications trade off between performance and delay should be considered.
  - Variations of the basic PCCC and SCCC is the hybridization, Hybrid concatenated Convolutional codes (HCCC).

![SCCC encoder block diagram](image)

**Figure 3** SCCC encoder block diagram
Concatenated codes and iterative decoding 5/7

SCCC Decoder structure

- The two decoders use a Soft Input Soft Output (SISO) module.

- The first decoder performs the decoding process and outputs soft decision that is de-interleaved and used by the second decoder as input coded symbol.

- The second decoder performs the decoding process and outputs soft decision that is interleaved and used by the first decoder as uncoded symbol again.

- The process is iterated many times before the final decision is made.

Figure 4 SCCC decoder block diagram
Concatenated codes and iterative decoding 6/7

- **SISO module**
  
  The key point in iterative decoding technique is the availability of a sub optimal algorithm that allows to decode the two codes separately and exchange the information between the two decoders.

- SISO module can be implemented by using MAP or SOVA

- MAP algorithm has better performance than SOVA but SOVA has less complexity than MAP.

- $P(u,I)$ probability of input information symbol
- $P(c,I)$ probability of input code symbol
- $P(u,O)$ probability of output information symbol
- $P(c,O)$ probability of output code symbol

Figure 5 The SISO module
Concatenated codes and iterative decoding 7/7

\[ P_t(c; O) = H_c \sum_{e : c(e) = c} A_{t-1}[S^c(e)]P_t[u(e); I]P_t[c(e); I]B_t[S^t(e)] \]

\[ P_t(u; O) = H_u \sum_{e : u(e) = u} A_{t-1}[S^u(e)]P_t[u(e); I]P_t[c(e); I]B_t[S^t(e)] \]

\[ A_t(s) = \sum_{e : S^e(e) = s} A_{t-1}[S^s(e)]P_t[u(e); I]P_t[c(e); I] \quad \text{for} \quad t = 1, ..., T \]

\[ B_t(s) = \sum_{e : S^e(e) = s} B_{t+1}[S^t(e)]P_t[u(e); I]P_{t+1}[c(e); I] \quad \text{for} \quad t = T - 1, ..., 0 \]

\[ H_c \rightarrow \sum_c P_t(c; O) = 1 \]

\[ H_u \rightarrow \sum_u P_t(u; O) = 1 \]

\[ A_0(s) = \langle 0^1 \rangle \quad s = s_0 \quad \text{otherwise} \]

\[ B_T(s) = \langle 0^1 \rangle \quad s = s_T \quad \text{otherwise} \]

**Figure 6** An edge of the trellis section
In fading channels the transmitted signal experiences severe magnitude fluctuations and phase rotation that makes it impossible to the receiver to determine the transmitted signal unless some less attenuated replica of the transmitted signal is provided to the receiver.

In space receive diversity two (or more) receive antenna are implemented at the receiver side.

- Space receive diversity is difficult to implement at the mobile station.

In space transmit diversity two (or more) antennas are implemented at the transmitter side.

Combining transmit diversity with channel coding creates a new family of codes known as Space-time codes.

- Space coding = Choosing an appropriate signal to transmit through each antenna
- Time coding = Channel code
Space-time codes 2/6

Basic structure of space-time codes
- At each time slot $t$, the output of modulator $i$ is a signal $c_i^t$ is transmitted using transmit antenna $i$.

Figure 7 Block diagram of the transmitter side
Space-time codes 3/6

Basic structure of space-time codes

At the receiver signal $d_t^j$ received by antenna $j$ at time $t$ is given by

$$d_t^j = \sum h_{i,j}^i c_i^i \sqrt{E_s} + n_t^i$$

where $h_{i,j}$ is the path gain from transmit antenna $i$ to receive antenna $j$.

$n_t^i$ is the noise at time $t$ which is modeled as independent samples of a zero mean complex Gaussian random variable with variance No/2 per dimension.

$\sqrt{E_s}$ is a factor that is chosen to make the average energy of the constellation to be 1.

Figure 8 Block diagram of the receiver side