S-38.211 Signal Processing in Communications	Stefan Werner
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Solution

a) The optimum coefficient vector is given by (LM 11.16)

$$\mathbf{c}_{opt} = \Phi^{-1}\mathbf{a} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix} = \frac{1}{1-0.5^2}\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}$$

$$= \frac{1}{0.75}\begin{bmatrix} 0.5-0.5\cdot 0.25 \\ -0.5\cdot 0.5+0.25 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

b) The minimum mean-square error (MMSE) is given by (LM 11.17)

$$\xi_{\min} = E\left[a_k^2\right] - \mathbf{a}^{\mathrm{T}}\Phi^{-1}\mathbf{a} = \sigma_a^2 - \mathbf{a}^{\mathrm{T}}\mathbf{c}_{opt} = \sigma_a^2 - \begin{bmatrix} 0.5 & 0.25 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0 \end{bmatrix} = \sigma_a^2 - 0.25$$

c) The MSEG algorithm is given by (LM 11-27)

$$\mathbf{c}_{j+1} = \mathbf{c}_j - \frac{\beta}{2}\nabla_{\mathbf{c}_j} E\left[\left|e_k\right|^2\right] = \mathbf{c}_j + \beta(\mathbf{a}-\Phi\mathbf{c}_j) = \mathbf{c}_j + \beta\begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix} - \beta\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}\mathbf{c}_j$$

d) The maximum step size that can be used is given by (LM 11-39) $0 < \beta < \frac{2}{\lambda_{\max}}$ where $\lambda_{\max}$ is the maximum eigenvalue to the autocorrelation matrix Φ.

$$\det(\lambda\mathbf{I}-\Phi) = 0 \quad \Leftrightarrow \quad \det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} \lambda-1 & -0.5 \\ -0.5 & \lambda-1 \end{bmatrix}\right) = (\lambda-1)^2 - 0.5^2 = 0$$

$$\Rightarrow \quad \lambda = 1 \pm 0.5 \quad \Rightarrow \quad \lambda_1 = 1.5, \lambda_2 = 0.5$$

We get the maximum step size as

$$\beta_{\max} = \frac{2}{\lambda_{\max}} = \frac{4}{3}.$$

Note that in order to be sure that the MSEG algorithm converges we choose a smaller step size than the above!