S-38.211 Signal Processing in Communications	Stefan Werner
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Figure 1 below shows an N tap FIR equalizer ($L = (N-1)/2$, N odd).

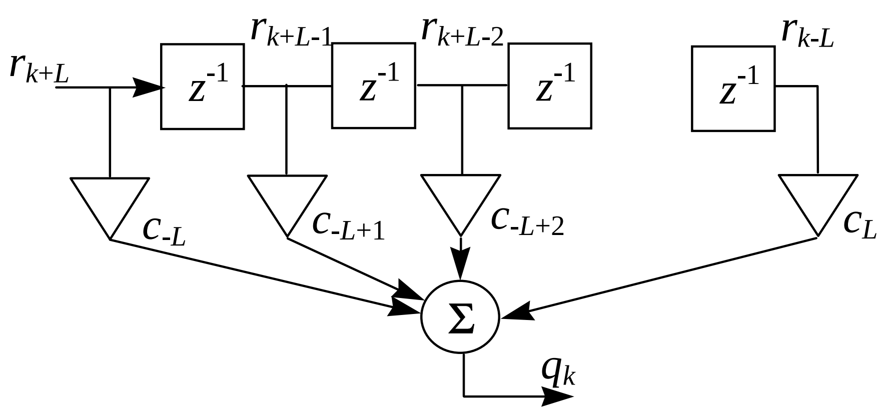

Figure 1: *N-tap FIR equalizer.*

Let us collect the filter coefficients and the input samples into the two vectors $\mathbf{c} = [c_{-L} \quad \cdots \quad c_0 \quad \cdots \quad c_L]^{\mathrm{T}}$ and $\mathbf{r}_k = [r_{k+L} \quad \cdots \quad r_k \quad \cdots \quad r_{k-L}]^{\mathrm{T}}$. We can now express the error between of the equalizer output as $e_k = a_k - q_k = a_k - \mathbf{c}^{\mathrm{T}}\mathbf{r}_k$, where a_k is the transmitted data q_k is the equalizer output.

The mean-squared error of the output is defined as $MSE = E\left[|e_k|^2\right] = E\left[|a_k - \mathbf{c}^{\mathrm{T}}\mathbf{r}_k|^2\right]$. The coefficient vector **c** that minimizes the *MSE* above is referred to as the optimum vector in the *mean-squared sense*. This solution is studied in LM-11.1.2. Several iterative algorithms exists that tries to solve this equation. One of them is the MSE gradient algorithm (LM Ch. 11.1.3) and it is the one we are going to take a look at in this exercise/homework.
(I think that before we start, I recapitulate some of the notations and formulas given in the class!!)

Exercise 4-1

Suppose the autocorrelation matrix Φ and the vector **a** are known for a given experimental environment.

$$\Phi = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \ \mathbf{a} = [0.5 \quad 0.25]^{\mathrm{T}}$$

a) Calculate the optimum coefficient vector (Wiener solution)

b) Calculate the minimum MSE considering that the variance of the transmitted signal $E\left[|a_k|^2\right]$ signal is given by σ_a^2.

c) For the environment described state the update formula for the MSEG algorithm.

d) What is the maximum step-size that can be used?

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Homework 4 (Return time: November 6, 1997)

Suppose the autocorrelation matrix Φ and the vector $\mathbf{a}$ are known for a given experimental environment.

$$\Phi = E\left[\mathbf{r}_k \mathbf{r}_k^{\mathrm{H}}\right] = \frac{1}{4}\begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 4 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} \frac{1}{2} & \frac{3}{8} & \frac{2}{8} & \frac{1}{8} \end{bmatrix}^{\mathrm{T}}$$

a) Calculate the optimum coefficient vector (Wiener solution)

b) Calculate the minimum MSE considering that the variance of the reference $E\left[|a_k|^2\right]$ signal is equal to one ($\sigma_a^2 = 1$).

c) For the environment described state the update formula for the MSEG algorithm.

d) Considering that the adaptive filter coefficients are initially zero, calculate their value for the ten first iterations.