S-38.211 Signal Processing in Communications	Stefan Werner
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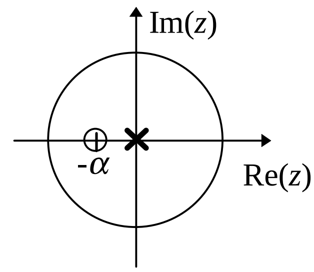
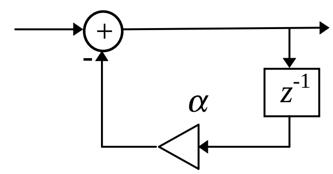
Solution

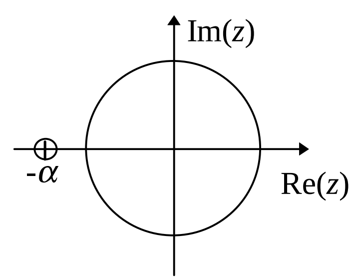
a) $S_C(z) = C(z)C(1/z^*)S_X(z) = (1+\alpha z^{-1})(1+\alpha z)\cdot 1$



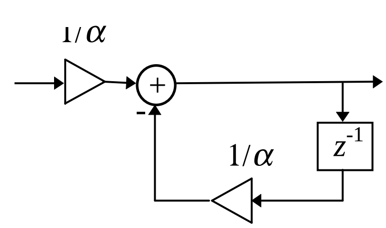
The channel is minimum-phase: $K(z) = \dfrac{1}{C(z)} = \dfrac{1}{1+\alpha z^{-1}}$ $\Rightarrow$



b) $S_C(z) = C(z)C(1/z^*)S_X(z) = (1+\alpha z^{-1})(1+\alpha z)\cdot 1 = \alpha\left(1+\dfrac{z}{\alpha}\right)\alpha\underbrace{\left(1+\dfrac{z^{-1}}{\alpha}\right)}_{\textit{Minimum-phase}}$



The equalizer is the part that is minimum-phase: $K(z) = \dfrac{\alpha^{-1}}{\left(1+\alpha^{-1}z^{-1}\right)}$ $\Rightarrow$



c)

$$\left|K_1(e^{j\omega})\right| = \left|\frac{1}{1+\alpha e^{-j\omega}}\right| = \frac{1}{\sqrt{(1+\alpha\cos\omega)^2 + (\alpha\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1+2\alpha\cos\omega+\alpha^2\cos^2\omega+\alpha^2\sin\omega^2}} = \frac{1}{\sqrt{1+2\alpha\cos\omega+\alpha^2}}$$

$$\left|K_2(e^{j\omega})\right| = \left|\frac{1}{\alpha+e^{-j\omega}}\right| = \frac{1}{\sqrt{(\alpha+\cos\omega)^2+\sin^2\omega^2}}$$

$$= \frac{1}{\sqrt{\alpha^2+2\alpha\cos\omega+\cos^2\omega+\sin\omega^2}} = \frac{1}{\sqrt{1+2\alpha\cos\omega+\alpha^2}}$$

d)

$$\sigma_C^2 = \int_{-\pi}^{\pi}\left|K_2(e^{j\omega})\right|^2 d\omega = \int_{-\pi}^{\pi}\frac{1}{1+2\alpha\cos\omega+\alpha^2}d\omega = \langle Parseval\rangle = \sum_{n=0}^{\infty}k_n^2 \qquad (*)$$

$$K(z) = \frac{1}{1+\alpha z^{-1}} = \sum_{n=0}^{\infty}(-\alpha z^{-1})^n = \sum_{n=0}^{\infty}(-\alpha)^n z^{-n} = \sum_{n=0}^{\infty}k_n z^{-n}$$

Now substituting $k_n = (-\alpha)^n$ into (*)

$$\sigma_C^2 = \sum_{n=0}^{\infty}\left(\alpha^2\right)^n = \frac{1}{1-\alpha^2}$$