Instructions and hints: This exercise contains quite simple probability calculations. Take your elementary probability math book (or similar reference) and try to find suitable formulas that should be applied. Just forget all telecom specific details and try to think what kind of random process is behind each example. Once you have found out suitable random process and formulas, the actual calculations should take only few minutes.

Note 1: bit error rate describes the probability of a bit being flipped, e.g., having an error. Usually it is expected (and you are encouraged to do so here) that the bit errors are uncorrelated and independent of each other.

Note 2: mathematical symbol to be used with rounding operations (instead of “Roundup()” etc.) are $\lfloor x \rfloor$ meaning biggest integer number less than $x$, i.e., truncation of the decimal part, and $\lceil x \rceil$ meaning smallest integer number greater than $x$, i.e., rounding-up.

1. (a) ATM cell header is protected by header error control (HEC) field which enables correction of single bit errors. What is the probability that there is uncorrectable errors, e.g., two or more incorrect bits, in ATM header when the bit error rate $p_{BER} = 10^{-4}$?

(b) In sub-marine transmission systems (G.975) as well as in OTN, forward error correction (FEC) coding is used. The FEC mechanism is based on Reed-Solomon (RS) codes and used coding is RS(255,239), i.e., 239 data bits are coded into 255 bit block (239 data bits + 16-bit parity). The degree $d$ of the coding polynomial is related to the number of parity symbols $r = d - 1$. An RS coding can correct up to $\lfloor (d-1)/2 \rfloor$ bit errors in a block. What is the probability that there is uncorrectable errors in a block when $p_{BER} = 10^{-5}$?

2. ATM cell delineation process is based on three state state-machine (see the figure below). The SYNC state is the normal operation state, HUNT state is used in start-up and error situations to find a correct cell header, and PRESYNC is used to make sure that the correct cell delineation is really found. In this case a correct header is considered to contain no bit errors.

(a) What is the probability of SYNC $\rightarrow$ HUNT transition, i.e., getting 7 consecutive incorrect HECs, when $p_{BER} = 10^{-4}$?
(b) How long, on the average, a 155.520 Mbit/s ATM system would remain in SYNC state?

3. Consider a cell switch with \( N \) input and output ports. In each time-slot at each input port, the probability that a cell arrives is \( p \). The incoming traffic is uniformly distributed, i.e., the probability that an incoming cell is destined to any output port is \( 1/N \).

   (a) What is the probability that \( k \) cells are destined to a particular output port?

   (b) What is the probability that more than one cell is destined to a particular output port?

4. (a) In certain transmission link 1 packet in every 1000 packets has an error. What is the probability that there are less than 7 invalid packets in a sample 8000 packets?

   (b) Certain switching unit is known to fail and unable to recover from error once is each 500 days on the average. What is the probability that the switching unit does not fail in 800 days? What is the probability that it fails within 200 days?