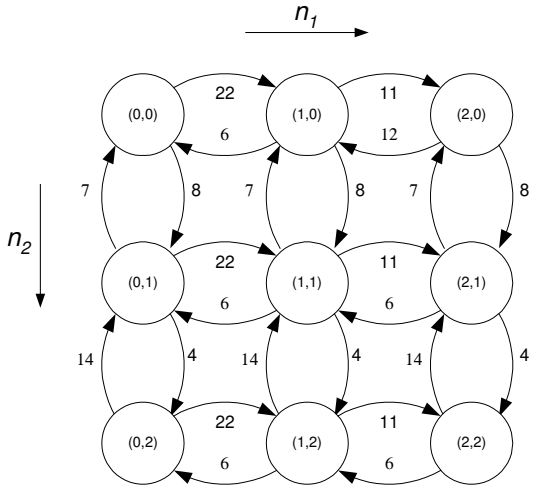
EXERCISE ABOUT EFFECTIVE BANDWIDTH

One of the models used to derive is based on a fluid flow approximation. This exercise focuses in more detail on the process generated from Markov modulated fluid flow sources. The article of Elwalid and Mitra on which this exercis is based is attached to this exercise.

Consider a multiplexing system which is fed by an aggregated fluid sources which is the superpostion of two types of ON/OFF fluid sources (type 1 and type 2). The behaviour of the aggregated fluid source is described by the following transition diagram:



Now the state of the continous-time Markov chain is given as $\mathbf{n} = (n_1, n_2)$. The stationary probability distribution of the entire multiplexing system is given by the 9 element vector $\boldsymbol{\pi}(x) = \{\pi_{s_i} | i\}$ with $\pi_{s_i}(x) = \Pr(s = s_i, X \le x)$, where the two-dimensional state vector $\mathbf{n} = (n_1, n_2)$ is mapped to the 1-dimensional one $s = s_i, \quad i = 1..9$ by $i = n_1 + 3n_2 + 1$. The governing system of differential equations for this multiplexing system is given as in equation (2) of Elwalid and Mitra.

1. Now the following questions relate to the situation described above.

a) How many sources of each type do we have (in other words, what are the values of N_1 and N_2)?

b) What are the average ON- and OFF-time for each source type?

c) Give the generator matrix $\mathbf{M}$ (sometimes this matrix is also called the $\mathbf{Q}$-matrix) for the Markov chain described by the transition diagram (Hint: the matrix elements m_{ij} of $\mathbf{M}$ are also known as the instantaneous transition rates).

2. Please refer to the article of Elwalid and Mitra (attached). What do the conditions $\mathbf{wM} = \mathbf{0}$ and $\langle \mathbf{w}, \mathbf{1} \rangle = 1$ (just before equation (3)) mean?

Reference:

A.I. Elwalid and D. Mitra, "Effective bandwidth of general Markovian traffic sources and admission control of high speed networks", *IEEE/ACM TNET*, Vol. 1, No. 3, pp. 329-343, June 1993