1. Decaying variance

Let $X_1, X_2, \ldots$ be i.i.d. random variables with finite mean and variance. Define aggregated process as

$$X_k^{(m)} = \frac{1}{m} (X_{km-m+1} + \ldots + X_{km}).$$

Prove that for $X_k^{(m)}$ it holds

$$V\left[ X_k^{(m)} \right] \sim \frac{a}{m},$$

where $a$ is some constant. (Process is said to have slowly decaying variance if $V\left[ X_k^{(m)} \right] \sim a m^{-\beta}, 0 < \beta < 1$ as $m \to \infty$)

2. Heavy-tailed distributions

Let random variable $X$ follow Pareto distribution that is

$$P\{X > x\} = \left( \frac{k}{x} \right)^\alpha, \quad x \geq k > 0 \text{ and } 0 < \alpha.$$

Prove that

(a) mean of $X$ is infinite if $\alpha \leq 1$
(b) variance of $X$ is also infinite if $\alpha < 2$

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1 independent identically distributed