Exercise 1

Consider a balanced allocation of the physical network model, characterized by \( \Phi(x) \). Show that the service capacities \( \psi_1, \ldots, \psi_N \) of the corresponding processor-sharing queueing network model are balanced by

\[
\Psi(y) = \prod_{i \in S_0} \frac{1}{y_i} \cdot \Phi(x) \prod_{k=1}^K \left( \frac{x_k}{y_k}, i \in S_k \right)
\]

Exercise 2

The following excerpt from [BP03a] contains an inconsistency. Find this inconsistency (1 point) and present a correction (1 point).

"4.1 Fair allocations

As mentioned in Section 1, most allocations considered so far in the literature are based on the notion of utility. Assume the utility of a flow is an increasing and strictly concave function \( U \) of its rate. A unique allocation is then defined by maximizing the overall utility:

\[
\sum_{k=1}^K x_k U \left( \frac{\phi_k(x)}{x_k} \right),
\]

under the capacity constraints (1). We say that these allocations are ”fair” in the sense that the utility function \( U \) is the same for all classes of flow. In particular, (...) The allocation associated with the log utility function is known as proportional fairness [14]. Another example is the range of allocations associated with the power functions \( U = (\cdots)^\alpha \), where the parameter \( \alpha, \alpha < 1, \alpha \neq 0 \), captures the trade-off between efficiency (in terms of overall allocated capacity \( \sum_{k=1}^K \phi_k(x) \)) and fairness. Specifically, the allocation maximizes the overall capacity when \( \alpha \to 1 \) and tends to max-min fairness when \( \alpha \to -\infty \) [23]. For convenience, we also refer to max-min fairness as a utility-based allocation.