

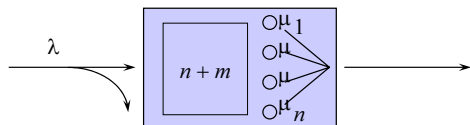
## 7. Loss systems

### Contents

- **Refresher: Simple teletraffic model**
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

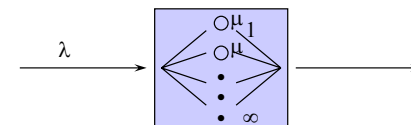
### Simple teletraffic model

- **Customers arrive** at rate  $\lambda$  (customers per time unit)
  - $1/\lambda$  = average inter-arrival time
- Customers are **served** by  $n$  parallel **servers**
- When busy, a server serves at rate  $\mu$  (customers per time unit)
  - $1/\mu$  = average service time of a customer
- There are  $n + m$  **customer places** in the system
  - at least  $n$  **service places** and at most  $m$  **waiting places**
- It is assumed that blocked customers (arriving in a full system) are lost



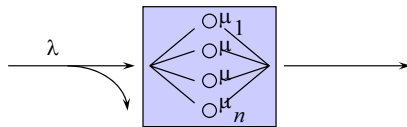
### Infinite system

- Infinite number of servers ( $n = \infty$ ), no waiting places ( $m = 0$ )
  - No customers are lost or even have to wait before getting served
- Sometimes,
  - this hypothetical model can be used to get some approximate results for a real system (with finite system capacity)
- Always,
  - it gives bounds for the performance of a real system (with finite system capacity)
  - it is much easier to analyze than the corresponding finite capacity models



## Pure loss system

- Finite number of servers ( $n < \infty$ ),  $n$  service places, no waiting places ( $m = 0$ )
  - If the system is full (with all  $n$  servers occupied) when a customer arrives, it is not served at all but lost
  - Some customers may be lost
- From the customer's point of view, it is interesting to know e.g.
  - What is the probability that the system is full when it arrives?



5

## Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

6

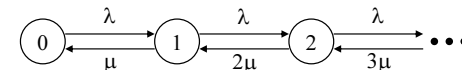
## Poisson model (M/M/ $\infty$ )

- **Definition: Poisson model** is the following simple teletraffic model:
  - Infinite number of independent customers ( $k = \infty$ )
  - Interarrival times are IID and exponentially distributed with mean  $1/\lambda$ 
    - so, customers arrive according to a Poisson process with intensity  $\lambda$
  - **Infinite** number of servers ( $n = \infty$ )
  - Service times are IID and exponentially distributed with mean  $1/\mu$
  - No waiting places ( $m = 0$ )
- Poisson model:
  - Using Kendall's notation, this is an M/M/ $\infty$  queue
  - Infinite system, and, thus, **lossless**
- Notation:
  - $a = \lambda/\mu =$  traffic intensity

7

## State transition diagram

- Let  $X(t)$  denote the number of customers in the system at time  $t$ 
  - Assume that  $X(t) = i$  at some time  $t$ , and consider what happens during a short time interval  $(t, t+h]$ :
    - with prob.  $\lambda h + o(h)$ , a new customer arrives (state transition  $i \rightarrow i+1$ )
    - if  $i > 0$ , then, with prob.  $i\mu h + o(h)$ , a customer leaves the system (state transition  $i \rightarrow i-1$ )
- Process  $X(t)$  is clearly a Markov process with state transition diagram



- Note that process  $X(t)$  is an irreducible birth-death process with an infinite state space  $S = \{0, 1, 2, \dots\}$

8

### Equilibrium distribution (1)

- Local balance equations (LBE):

$$\pi_i \lambda = \pi_{i+1} (i+1) \mu \quad (\text{LBE})$$

$$\Rightarrow \pi_{i+1} = \frac{\lambda}{(i+1)\mu} \pi_i = \frac{a}{i+1} \pi_i$$

$$\Rightarrow \pi_i = \frac{a^i}{i!} \pi_0, \quad i = 0, 1, 2, \dots$$

- Normalizing condition (N):

$$\sum_{i=0}^{\infty} \pi_i = \pi_0 \sum_{i=0}^{\infty} \frac{a^i}{i!} = 1 \quad (\text{N})$$

$$\Rightarrow \pi_0 = \left( \sum_{i=0}^{\infty} \frac{a^i}{i!} \right)^{-1} = (e^a)^{-1} = e^{-a}$$

9

### Equilibrium distribution (2)

- Thus, the equilibrium distribution is a **Poisson distribution**:

$$X \sim \text{Poisson}(a)$$

$$P\{X = i\} = \pi_i = \frac{a^i}{i!} e^{-a}, \quad i = 0, 1, 2, \dots$$

$$E[X] = a, \quad D^2[X] = a$$

- Remark:** Insensitivity with respect to service time distribution
  - The result is **insensitive** to the service time distribution, that is: it is valid for **any** service time distribution with mean  $1/\mu$
  - So, instead of the M/M/ $\infty$  model, we can consider, as well, the more general M/G/ $\infty$  model

10

### Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

11

### Application to flow level modelling of streaming data traffic

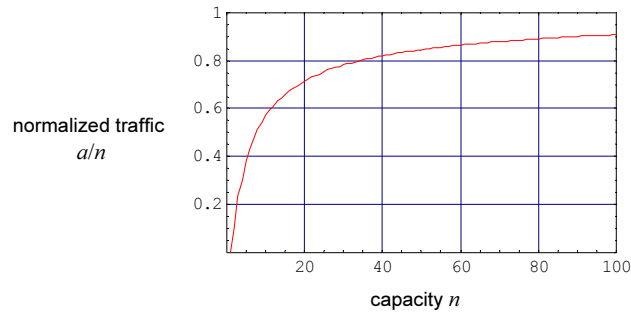
- Poisson model may be applied to flow level modelling of streaming data traffic
  - customer = UDP flow with constant bit rate (CBR)
  - $\lambda$  = flow arrival rate (flows per time unit)
  - $h = 1/\mu$  = average flow duration (time units)
  - $a = \lambda\mu$  = traffic intensity
  - $r$  = bit rate of a flow (data units per time unit)
  - $N$  = nr of active flows obeying Poisson( $a$ ) distribution
- When the total transmission rate  $Nr$  exceeds the link capacity  $C = nr$ , bits are lost
  - loss ratio**  $p_{\text{loss}}$  gives the ratio between the traffic lost and the traffic offered:

$$p_{\text{loss}} = \frac{E[(Nr - C)^+]}{E[Nr]} = \frac{E[(N - n)^+]}{E[N]} = \frac{1}{a} \sum_{i=n+1}^{\infty} (i - n) \frac{a^i}{i!} e^{-a}$$

12

## Multiplexing gain

- We determine traffic intensity  $a$  so that loss ratio  $p_{\text{loss}} < 1\%$
- Multiplexing gain** is described by the traffic intensity per capacity unit,  $a/n$ , as a function of capacity  $n$



13

## Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)**
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

14

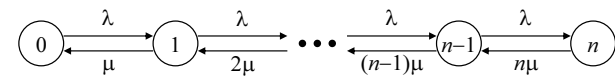
## Erlang model (M/M/n/n)

- Definition: Erlang model** is the following simple teletraffic model:
  - Infinite number of independent customers ( $k = \infty$ )
  - Interarrival times are IID and exponentially distributed with mean  $1/\lambda$ 
    - so, customers arrive according to a Poisson process with intensity  $\lambda$
  - Finite** number of servers ( $n < \infty$ )
  - Service times are IID and exponentially distributed with mean  $1/\mu$
  - No waiting places ( $m = 0$ )
- Erlang model:
  - Using Kendall's notation, this is an M/M/n/n queue
  - Pure loss system, and, thus, **lossy**
- Notation:
  - $a = \lambda/\mu =$  traffic intensity

15

## State transition diagram

- Let  $X(t)$  denote the number of customers in the system at time  $t$ 
  - Assume that  $X(t) = i$  at some time  $t$ , and consider what happens during a short time interval  $(t, t+h]$ :
    - with prob.  $\lambda h + o(h)$ , a new customer arrives (state transition  $i \rightarrow i+1$ )
    - with prob.  $i\mu h + o(h)$ , a customer leaves the system (state transition  $i \rightarrow i-1$ )
- Process  $X(t)$  is clearly a Markov process with state transition diagram



- Note that process  $X(t)$  is an irreducible birth-death process with a finite state space  $S = \{0, 1, 2, \dots, n\}$

16

### Equilibrium distribution (1)

- Local balance equations (LBE):

$$\pi_i \lambda = \pi_{i+1} (i+1) \mu \quad (\text{LBE})$$

$$\Rightarrow \pi_{i+1} = \frac{\lambda}{(i+1)\mu} \pi_i = \frac{a}{i+1} \pi_i$$

$$\Rightarrow \pi_i = \frac{a^i}{i!} \pi_0, \quad i = 0, 1, \dots, n$$

- Normalizing condition (N):

$$\sum_{i=0}^n \pi_i = \pi_0 \sum_{i=0}^n \frac{a^i}{i!} = 1 \quad (\text{N})$$

$$\Rightarrow \pi_0 = \left( \sum_{i=0}^n \frac{a^i}{i!} \right)^{-1}$$

17

### Equilibrium distribution (2)

- Thus, the equilibrium distribution is a **truncated Poisson distribution**:

$$P\{X = i\} = \pi_i = \frac{\frac{a^i}{i!}}{\sum_{j=0}^n \frac{a^j}{j!}}, \quad i = 0, 1, \dots, n$$

- Remark:** Insensitivity with respect to the service time distribution
  - The result is **insensitive** to the service time distribution, that is: it is valid for **any** service time distribution with mean  $1/\mu$
  - So, instead of the  $M/M/n/n$  model, we can consider, as well, the more general  $M/G/n/n$  model

18

### Time blocking

- Time blocking**  $B_t$  = probability that all  $n$  servers are occupied at an arbitrary time = the fraction of time that all  $n$  servers are occupied
- For a stationary Markov process, this equals the probability  $\pi_n$  of the equilibrium distribution  $\pi$ . Thus,

$$B_t := P\{X = n\} = \pi_n = \frac{\frac{a^n}{n!}}{\sum_{j=0}^n \frac{a^j}{j!}}$$

19

### Call blocking

- Call blocking**  $B_c$  = probability that an arriving customer finds all  $n$  servers occupied = the fraction of arriving customers that are lost
- However, due to Poisson arrivals and PASTA property, the probability that an arriving customer finds all  $n$  servers occupied equals the probability that all  $n$  servers are occupied at an arbitrary time,
- In other words, call blocking  $B_c$  equals time blocking  $B_t$ :

$$B_c = B_t = \frac{\frac{a^n}{n!}}{\sum_{j=0}^n \frac{a^j}{j!}}$$

- This is **Erlang's blocking formula**

20

## Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

21

## Application to telephone traffic modelling in trunk network

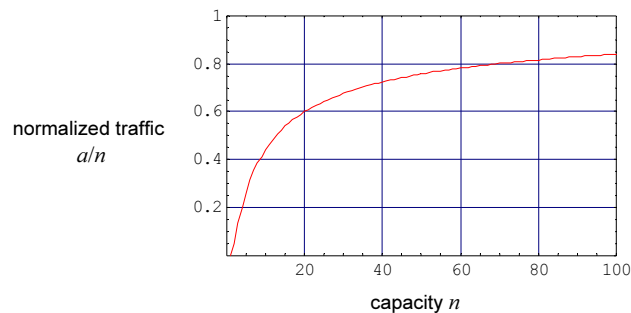
- Erlang model may be applied to modelling of telephone traffic in trunk network where the number of potential users of a link is large
  - customer = call
  - $\lambda$  = call arrival rate (calls per time unit)
  - $h = 1/\mu$  = average call holding time (time units)
  - $a = \lambda h = \lambda/\mu$  = traffic intensity
  - $n$  = link capacity (channels)
- A call is lost if all  $n$  channels are occupied when the call arrives
  - **call blocking**  $B_c$  gives the probability of such an event

$$B_c = \frac{\frac{a^n}{n!}}{\sum_{j=0}^n \frac{a^j}{j!}}$$

22

## Multiplexing gain

- We determine traffic intensity  $a$  so that call blocking  $B_c < 1\%$
- **Multiplexing gain** is described by the traffic intensity per capacity unit,  $a/n$ , as a function of capacity  $n$



23

## Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

24

## Binomial model (M/M/k/k/k)

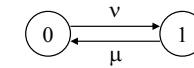
- **Definition: Binomial model** is the following (simple) teletraffic model:
  - **Finite** number of independent customers ( $k < \infty$ )
    - **on-off type** customers (alternating between idleness and activity)
  - Idle times are IID and exponentially distributed with mean  $1/\nu$
  - As many servers as customers ( $n = k$ )
  - Service times are IID and exponentially distributed with mean  $1/\mu$
  - No waiting places ( $m = 0$ )
- Binomial model:
  - Using Kendall's notation, this is an M/M/k/k/k queue
  - Although a finite system, this is clearly **lossless**
- On-off type customer:



25

## On-off type customer (1)

- Let  $X_j(t)$  denote the state of customer  $j$  ( $j = 1, 2, \dots, k$ ) at time  $t$ 
  - State 0 = idle, state 1 = active = in service
  - Consider what happens during a short time interval  $(t, t+h]$ :
    - if  $X_j(t) = 0$ , then, with prob.  $\nu h + o(h)$ , the customer becomes active (state transition  $0 \rightarrow 1$ )
    - if  $X_j(t) = 1$ , then, with prob.  $\mu h + o(h)$ , the customer becomes idle (state transition  $1 \rightarrow 0$ )
- Process  $X_j(t)$  is clearly a Markov process with state transition diagram



- Note that process  $X_j(t)$  is an irreducible birth-death process with a finite state space  $S = \{0, 1\}$

26

## On-off type customer (2)

- Local balance equations (LBE):

$$\pi_0^{(j)} \nu = \pi_1^{(j)} \mu \Rightarrow \pi_1^{(j)} = \frac{\nu}{\mu} \pi_0^{(j)}$$

- Normalizing condition (N):

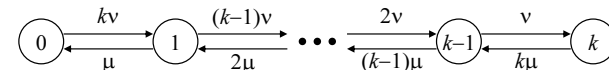
$$\pi_0^{(j)} + \pi_1^{(j)} = \pi_0^{(j)} \left(1 + \frac{\nu}{\mu}\right) = 1 \Rightarrow \pi_0^{(j)} = \frac{\mu}{\nu + \mu}, \quad \pi_1^{(j)} = \frac{\nu}{\nu + \mu}$$

- So, the equilibrium distribution of a single customer is the **Bernoulli distribution** with success probability  $\nu/(\nu + \mu)$ 
  - offered traffic is  $\nu/(\nu + \mu)$
- From this, we could deduce that the equilibrium distribution of the state of the whole system (that is: the number of active customers) is the binomial distribution  $\text{Bin}(k, \nu/(\nu + \mu))$

27

## State transition diagram

- Let  $X(t)$  denote the number of active customers
  - Assume that  $X(t) = i$  at some time  $t$ , and consider what happens during a short time interval  $(t, t+h]$ :
    - if  $i < k$ , then, with prob.  $(k-i)\nu h + o(h)$ , an idle customer becomes active (state transition  $i \rightarrow i+1$ )
    - if  $i > 0$ , then, with prob.  $i\mu h + o(h)$ , an active customer becomes idle (state transition  $i \rightarrow i-1$ )
- Process  $X(t)$  is clearly a Markov process with state transition diagram



- Note that process  $X(t)$  is an irreducible birth-death process with a finite state space  $S = \{0, 1, \dots, k\}$

28

### Equilibrium distribution (1)

- Local balance equations (LBE):

$$\pi_i(k-i)v = \pi_{i+1}(i+1)\mu \quad (\text{LBE})$$

$$\Rightarrow \pi_{i+1} = \frac{(k-i)v}{(i+1)\mu} \pi_i$$

$$\Rightarrow \pi_i = \frac{k!}{i!(k-i)!} \left(\frac{v}{\mu}\right)^i \pi_0 = \binom{k}{i} \left(\frac{v}{\mu}\right)^i \pi_0, \quad i = 0, 1, \dots, k$$

- Normalizing condition (N):

$$\sum_{i=0}^k \pi_i = \pi_0 \sum_{i=0}^k \binom{k}{i} \left(\frac{v}{\mu}\right)^i = 1 \quad (\text{N})$$

$$\Rightarrow \pi_0 = \left( \sum_{i=0}^k \binom{k}{i} \left(\frac{v}{\mu}\right)^i \right)^{-1} = \left(1 + \frac{v}{\mu}\right)^{-k} = \left(\frac{\mu}{v+\mu}\right)^k$$

### Equilibrium distribution (2)

- Thus, the equilibrium distribution is a **binomial distribution**:

$$X \sim \text{Bin}\left(k, \frac{v}{v+\mu}\right)$$

$$P\{X = i\} = \pi_i = \binom{k}{i} \left(\frac{v}{v+\mu}\right)^i \left(\frac{\mu}{v+\mu}\right)^{k-i}, \quad i = 0, 1, \dots, k$$

$$E[X] = \frac{kv}{v+\mu}, \quad D^2[X] = k \cdot \frac{v}{v+\mu} \cdot \frac{\mu}{v+\mu} = \frac{kv\mu}{(v+\mu)^2}$$

- Remark:** Insensitivity w.r.t. service time and idle time distribution
  - The result is **insensitive both** to the service **and** the idle time distribution, that is: it is valid for **any** service time distribution with mean  $1/\mu$  and **any** idle time distribution with mean  $1/v$
  - So, instead of the M/M/k/k model, we can consider, as well, the more general G/G/k/k model

### Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

### Engset model (M/M/n/n/k)

- Definition:** Engset model is the following (simple) teletraffic model:

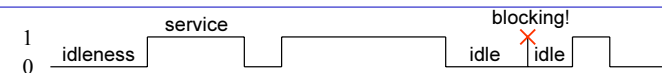
- Finite** number of independent customers ( $k < \infty$ )
  - on-off type** customers (alternating between idleness and activity)
- Idle times are IID and exponentially distributed with mean  $1/v$
- Less servers** than customers ( $n < k$ )
- Service times are IID and exponentially distributed with mean  $1/\mu$
- No waiting places ( $m = 0$ )

- Engset model:

- Using Kendall's notation, this is an M/M/n/n/k queue
- This is a pure loss system, and, thus, **lossy**

- On-off type customer:

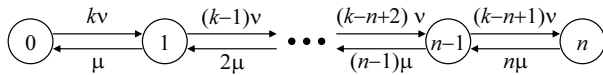
Note: If the system is full when an idle cust. tries to become an active cust., a new idle period starts.





## State transition diagram

- Let  $X(t)$  denote the number of active customers
  - Assume that  $X(t) = i$  at some time  $t$ , and consider what happens during a short time interval  $(t, t+h]$ :
    - if  $i < n$ , then, with prob.  $(k-i)v h + o(h)$ , an idle customer becomes active (state transition  $i \rightarrow i+1$ )
    - if  $i > 0$ , then, with prob.  $i\mu h + o(h)$ , an active customer becomes idle (state transition  $i \rightarrow i-1$ )
- Process  $X(t)$  is clearly a Markov process with state transition diagram



- Note that process  $X(t)$  is an irreducible birth-death process with a finite state space  $S = \{0, 1, \dots, n\}$

33

## Equilibrium distribution (1)

- Local balance equations (LBE):

$$\pi_i (k-i)v = \pi_{i+1} (i+1)\mu \quad (\text{LBE})$$

$$\Rightarrow \pi_{i+1} = \frac{(k-i)v}{(i+1)\mu} \pi_i$$

$$\Rightarrow \pi_i = \frac{k!}{i!(k-i)!} \left(\frac{v}{\mu}\right)^i \pi_0 = \binom{k}{i} \left(\frac{v}{\mu}\right)^i \pi_0, \quad i = 0, 1, \dots, n$$

- Normalizing condition (N):

$$\sum_{i=0}^n \pi_i = \pi_0 \sum_{i=0}^n \binom{k}{i} \left(\frac{v}{\mu}\right)^i = 1 \quad (\text{N})$$

$$\Rightarrow \pi_0 = \left( \sum_{i=0}^n \binom{k}{i} \left(\frac{v}{\mu}\right)^i \right)^{-1}$$

34

## Equilibrium distribution (2)

- Thus, the equilibrium distribution is a **truncated binomial distribution**:

$$P\{X = i\} = \pi_i = \frac{\binom{k}{i} \left(\frac{v}{\mu}\right)^i}{\sum_{j=0}^n \binom{k}{j} \left(\frac{v}{\mu}\right)^j} = \frac{\binom{k}{i} \left(\frac{v}{v+\mu}\right)^i \left(\frac{\mu}{v+\mu}\right)^{k-i}}{\sum_{j=0}^n \binom{k}{j} \left(\frac{v}{v+\mu}\right)^j \left(\frac{\mu}{v+\mu}\right)^{k-j}}, \quad i = 0, \dots, n$$

- Offered traffic is  $kv/(v+\mu)$
- Remark:** Insensitivity w.r.t. service time and idle time distribution
  - The result is **insensitive both** to the service **and** the idle time distribution, that is: it is valid for **any** service time distribution with mean  $1/\mu$  and **any** idle time distribution with mean  $1/v$
  - So, instead of the M/M/n/n/k model, we can consider, as well, the more general G/G/n/n/k model

35

## Time blocking

- Time blocking**  $B_t$  = probability that all  $n$  servers are occupied at an arbitrary time = the fraction of time that all  $n$  servers are occupied
- For a stationary Markov process, this equals the probability  $\pi_n$  of the equilibrium distribution  $\pi$ . Thus,

$$B_t := P\{X = n\} = \pi_n = \frac{\binom{k}{n} \left(\frac{v}{\mu}\right)^n}{\sum_{j=0}^n \binom{k}{j} \left(\frac{v}{\mu}\right)^j}$$

36

### Call blocking (1)

- **Call blocking**  $B_c$  = probability that an arriving customer finds all  $n$  servers occupied = the fraction of arriving customers that are lost
  - In the Engset model, however, the “arrivals” do **not** follow a Poisson process. Thus, we cannot utilize the PASTA property any more.
  - In fact, the distribution of the state that an “arriving” customer sees differs from the equilibrium distribution. Thus, call blocking  $B_c$  does **not** equal time blocking  $B_t$  in the Engset model.

37

### Call blocking (2)

- Let  $\pi_i^*$  denote the probability that there are  $i$  active customers when an idle customer becomes active (which is called an “arrival”)
- Consider a long time interval  $(0, T)$ :
  - During this interval, the average time spent in state  $i$  is  $\pi_i T$
  - During this time, the average number of “arriving” customers (who all see the system to be in state  $i$ ) is  $(k-i)v \cdot \pi_i T$
  - During the whole interval, the average number of “arriving” customers is  $\sum_j (k-j)v \cdot \pi_j T$
- Thus,

$$\pi_i^* = \frac{(k-i)v \cdot \pi_i T}{\sum_{j=0}^n (k-j)v \cdot \pi_j T} = \frac{(k-i) \cdot \pi_i}{\sum_{j=0}^n (k-j) \cdot \pi_j}, \quad i = 0, 1, \dots, n$$

38

### Call blocking (3)

- It can be shown (exercise!) that

$$\pi_i^* = \frac{\binom{k-1}{i} \left(\frac{v}{\mu}\right)^i}{\sum_{j=0}^n \binom{k-1}{j} \left(\frac{v}{\mu}\right)^j}, \quad i = 0, 1, \dots, n$$

- If we write explicitly the dependence of these probabilities on the total number of customers, we get the following result:

$$\pi_i^*(k) = \pi_i(k-1), \quad i = 0, 1, \dots, n$$

- In other words, an “arriving” customer sees such a system where there is one customer less (itself!) in equilibrium

39

### Call blocking (4)

- By choosing  $i = n$ , we get the following formula for the call blocking probability:

$$B_c(k) = \pi_n^*(k) = \pi_n(k-1) = B_t(k-1)$$

- Thus, for the Engset model, the call blocking in a system with  $k$  customers equals the time blocking in a system with  $k-1$  customers:

$$B_c(k) = B_t(k-1) = \frac{\binom{k-1}{n} \left(\frac{v}{\mu}\right)^n}{\sum_{j=0}^n \binom{k-1}{j} \left(\frac{v}{\mu}\right)^j}$$

- This is **Engset's blocking formula**

40

## Contents

- Refresher: Simple teletraffic model
- Poisson model ( $\infty$  customers,  $\infty$  servers)
- Application to flow level modelling of streaming data traffic
- Erlang model ( $\infty$  customers,  $n < \infty$  servers)
- Application to telephone traffic modelling in trunk network
- Binomial model ( $k < \infty$  customers,  $n = k$  servers)
- Engset model ( $k < \infty$  customers,  $n < k$  servers)
- Application to telephone traffic modelling in access network

41

## Application to telephone traffic modelling in access network

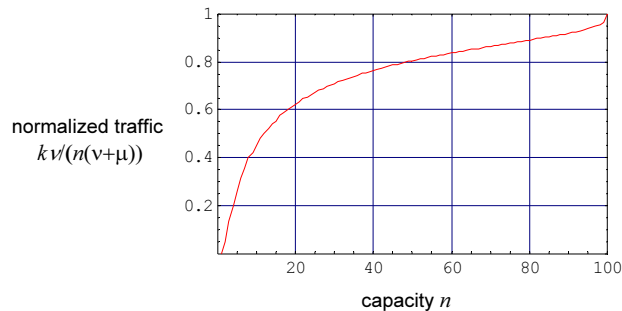
- Engset model may be applied to modelling of telephone traffic in access network where the nr of potential users of a link is moderate
  - customer = call
  - $\nu$  = call arrival rate per idle user (calls per time unit)
  - $1/\mu$  = average call holding time (time units)
  - $k$  = number of potential users
  - $n$  = link capacity (channels)
- A call is lost if all  $n$  channels are occupied when the call arrives
  - **call blocking**  $B_c$  gives the probability of such an event

$$B_c = \frac{\binom{k-1}{n} \left(\frac{\nu}{\mu}\right)^n}{\sum_{j=0}^n \binom{k-1}{j} \left(\frac{\nu}{\mu}\right)^j}$$

42

## Multiplexing gain

- We assume that an access link is loaded by  $k = 100$  potential users
- We determine traffic intensity  $k\nu/(\nu+\mu)$  so that call blocking  $B_c < 1\%$
- **Multiplexing gain** is described by the traffic intensity per capacity unit,  $k\nu/(n(\nu+\mu))$ , as a function of capacity  $n$



43