HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.145 Introduction to Teletraffic Theory, Spring 2005

Exercise 11
21.4.2005

Problems 2–3 are homework exercises. Mark the problems you have solved in the beginning of the exercise class.

1. *Demo*
   Simulate, according to the discrete event simulation principles presented in the lectures, the evolution of the queue length process $Q(t)$ (those waiting to be serviced and the one being serviced) in an M/M/1-FIFO queue during the interval $[0, T]$ assuming that the system is empty in the beginning ($Q(0) = 0$). Let $\lambda = 1/2$, $\mu = 1$, and $T = 2000$. Make $n = 100$ independent simulation runs. Independent means that the seed value for the random number generation changes. In each simulation run, calculate the mean queue length X in the interval $[T_0, T]$, where $T_0 = 1000$, from the equation

$$X = \frac{1}{T - T_0} \int_{T_0}^{T} Q(t)dt.$$

   By this way, you get n IID samples $X_1, X_2, \ldots, X_n$ of the mean queue length in this interval.

   a) Calculate and plot the sample average $\bar{X}_m$, for $m = 10, 20, \ldots, n$,

$$\bar{X}_m = \frac{1}{m} \sum_{i=1}^{m} X_i.$$

   b) Calculate and plot the square root of the sample variance S_m, for $m = 10, 20, \ldots, n$,

$$S_m = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (X_i - \bar{X}_m)^2}.$$

   c) Calculate and plot the confidence interval for the sample average $\bar{X}_m$ at confidence level 95% for $m = 10, 20, \ldots, n$, assuming that the samples are IID and from a normal distribution, but with an unknown variance.

2. *Homework exercise (2 points)*
   Generation of random numbers.

   (a) Generate 4 (pseudo) random numbers from $U(0, 1)$ distribution using the MCG-algorithm presented in the lectures (Lecture 11, slide 24) with parameters $m = 2^{31} - 1$, $a = 16,807$ and $Z_0 = 123456$.

   (b) Utilizing the random numbers generated in part (a), generate 4 random number from each of the following distributions; $U(2, 4)$, $Bin(3, 0.2)$, $Exp(2)$, $N(2, 3)$. Use the methods described in the lectures.

3. *Homework exercise (1 point)*
   Assume the simulation runs have yielded the following measurements for a performance parameter α: 6.59, 3.50, 1.95, 3.98, 2.36. Calculate 95% confidence interval for sample mean $\bar{X}_n$

   (a) assuming that the variance is known ($\mathrm{Var}[X_i] = 2$).

   (b) assuming that the variance is not known.