HELSINKI UNIVERSITY OF TECHNOLOGY
Networking Laboratory
S-38.145 Introduction to Teletraffic Theory, Spring 2005

Exercise 4
24.2.2005

Problems 2–3 are homework exercises. Mark the problems you have solved in the beginning of the exercise class.

1. *Demo*
   Let X and Y be independent random variables. Consider then the random variable $Z = aX + bY$, where a, b are real numbers

   (a) Determine the mean and variance of Z.

   (b) Assume that X ∼Poisson(3) and Y ∼Poisson(2), $a = b = 5$. What is the probability $\mathrm{P}(Z = 0)$?

2. *Homework exercise (1 point)*
   Suppose that the lifetime X of a fuse (in 100 hour units) is exponentially distributed with $\mathrm{P}(X > 10) = 0.8$.

   (a) Determine the rate parameter λ.

   (b) Determine the mean and variance of X.

   (c) Determine the median of the lifetime, i.e. t for which $\mathrm{P}(X > t) = 0.5$.

3. *Homework exercise (1 point)*
   Suppose that you are participating in a game show and the show's host brings three identical boxes in front of you. One of the boxes contains a fabulous prize and the other two are just empty. The host asks you to try to win the prize by choosing one of the boxes. After you have indicated your decision to the host, he opens up one of the two other boxes you did not select, which he knows to be empty (since only one contains a prize, at least one of the remaining boxes is empty). There are now two boxes remaining; the one you have chosen and another, and the host asks you whether you want to change your box. What is the best policy to get the prize, stick with your original choice, change your mind, or does the policy play any role at all in the outcome of the game?

   Tip: One way to solve the problem is to look at the conditional probabilities. Without loss of generality, we can say that you have chosen box A and the host opens box B. Calculate the conditional probabilities for the cases that the prize is in A and in C, respectively, on condition that the host has opened B. Utilize Bayes' rule and figure out the required probabilities.