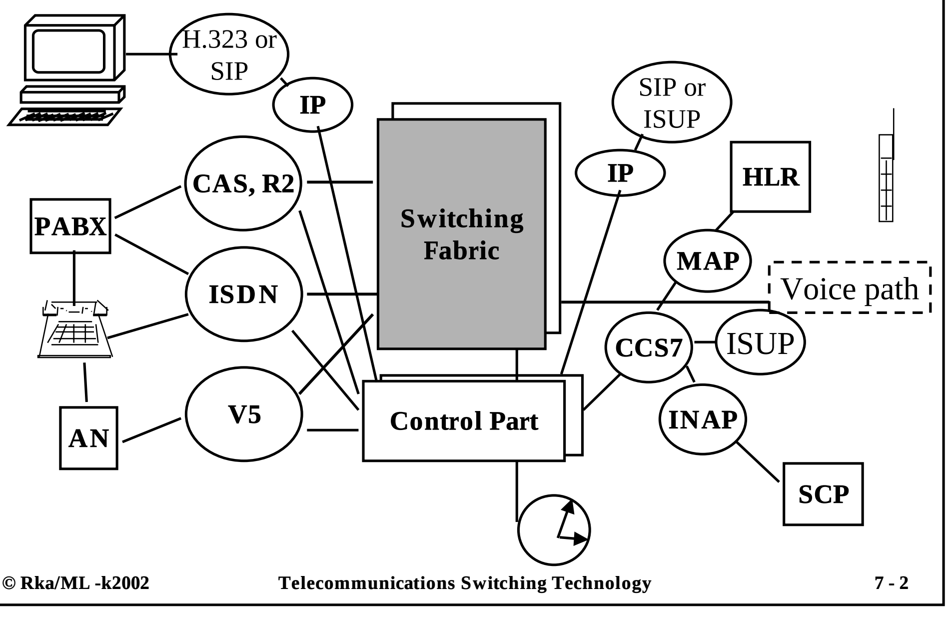
Switching Fabrics - Recursion, Cantor network

- ✔ **Some repetition**
- ✔ **Non-blocking property**
  - **rearrangement of existing thruconnections**
  - **strict sense non-blocking fabric**
- ✔ **Generic three stage switch fabric**
- ✔ **Clos network**
- ✔ **Benes network**
- ✔ **Cantor network**
- ✔ **Cross-points and cross-point complexity**

Summary of course scope

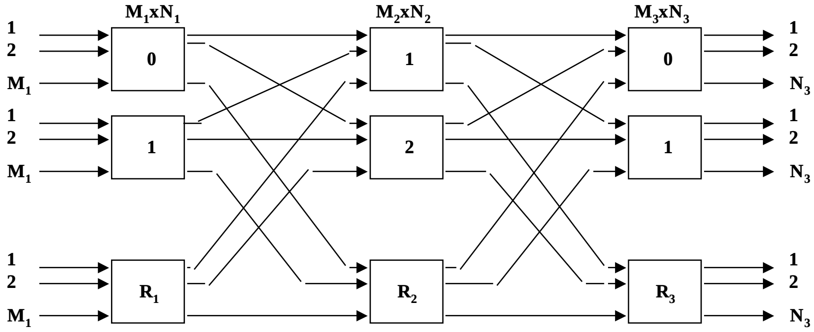


Characteristic properties of switch fabrics

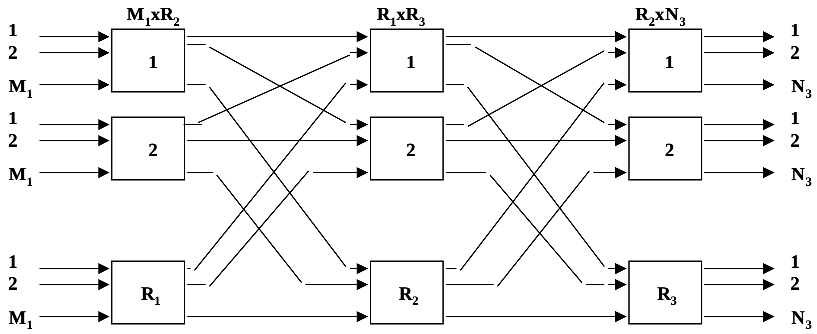
- ✔ **Nrof cross-points in a fabric is the nrof "and-gates" in an equivalent space switch.**
- ✔ **Logical depth is the nrof cross-points on the signal path thru the fabric.**
- ✔ **Blocking probability is a function of the structure of the switch fabric**
- ✔ **Fan-out requirement of the cross-point is the nrof inputs the output of our cp needs to feed.**

A generic representation of a three stage switch fabric

- ✔ **A Three stage switch fabric, logically reduced into a network of space switching blocks, has connections from each block to each block of the following stage.**



Clos Network is a special case of our generic three stage switch fabric



- **Stage 1: $N_1 = R_2$**
- **Stage 2: $M_2 = R_1$ ja $N_2 = R_3$**
- **Stage 3: $M_3 = R_2$**

E.g. 8192 PCM, 16M = 8 x 2M base rate:
$M_1 = N_3 = 1024$,
$R_1 = R_3 = 8*32 = 256$,
$R_2 = 2048$ ---> strict sense non-blocking

When input/output signals and the capacity of the fabric are given, the only free variable is $\mathbf{R_2}$

Strict sense non-blocking Clos network

- ✔ **A Clos network is strict sense non-blocking, when the number of switch blocks in the second stage is**

$$R_2 => M_1 + N_3 - 1$$

- ✔ **In the special case of a symmetric switch fabric, i.e. for $M_1 = N_3 = N$**

$$R_2 => 2N - 1$$

Rearrangeably non-blocking Clos network

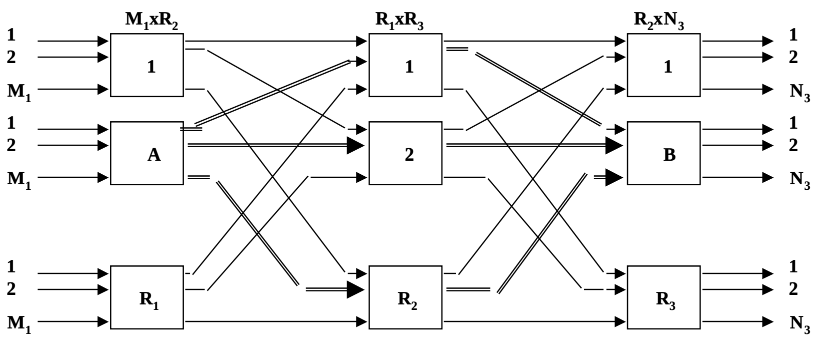
- ✔ **A three stage Clos network is rearrangeably non-blocking, when**

$$R_2 => \max(M_1 , N_3)$$

- ✔ **In the special case of a symmetric switch fabric, i.e. for $M_1 = N_3 = N$**

$$R_2 => N$$

Thruconnection from block A to block B



For a thruconnection, there are R_2 alternative paths in a fabric with no other established thruconnections.

Clos theorem

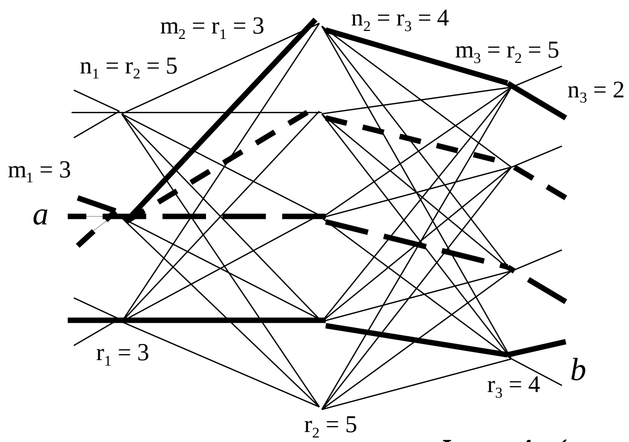
A Clos network is strict sense non-blocking, íf and only if the nrof switch blocks in the second stage is $r_2 \geq m_1 + n_3 - 1$.

In particular, a symmetric Clos network with $m_1 = n_3 = n$, is strict sense non-blocking if and only if $r_2 \geq 2n - 1$.

Proof: We will use the Paull's matrix representation. Assume pt-to-pt connections only!
- Assume row *a* with a free input and column *b* with a free output
- mark the thruconnection of the free input to the free output with a new symbol in (*a*, *b*)
- on row *a* there are max $m_1 - 1$ different symbols because *a* has m_1 inputs
- column *b* has max $n_3 - 1$ different symbols
- at worst, together there are max $m_1 - 1 + n_3 - 1$ different symbols
- if we have one more unused switch block, i.e. $m_1 + n_3 - 1$ in total, thruconnection can be established

Necessity: The following thruconnections should be possible:
- **total of m_1 thruconnections from *a* distributed to all third stage switches (each time a different symbols must be used)**
- **to *b* from each first stage switch, except *a*: total of $n_3 - 1$ (each time a diff symbol), i.e.**
- **on row *a* and column *b* the total of $m_1 + n_3 - 1$ different symbols are required!**

Visualization of the necessity of the CLOS requirement

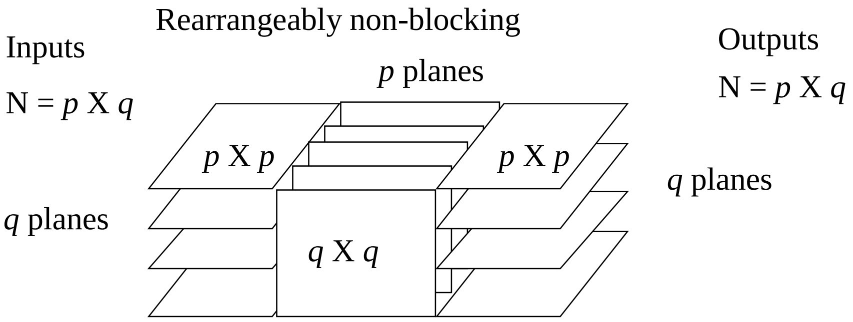


I.e $r_2 = 4 = (m_1 + n_3 -1)$ is necessary and sufficient!

Alternative representations of a switch fabric

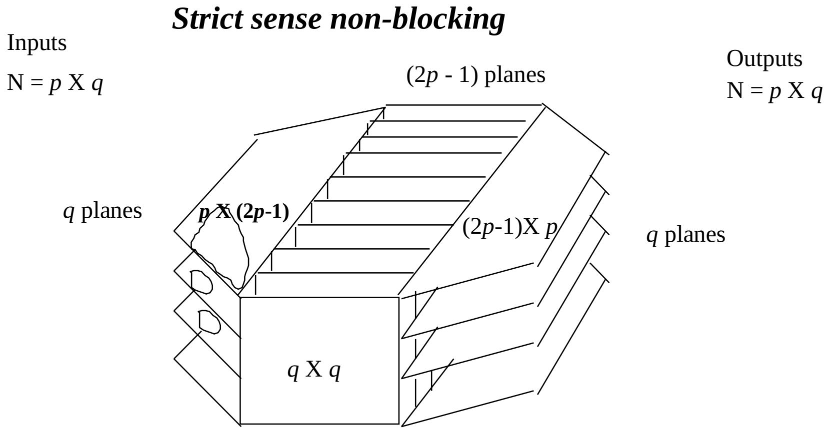
- ✔ **A Fabric can be represented either using the generic representation, using horizontal and vertical planes, or using a graph.**
- ✔ **When planes are used, we avoid showing the connections as lines - they are replaced by touch-points of the planes.**
- ✔ **A graph is good at visualizing thruconnections and the use of stage to stage lines only once.**

Recursive construction of a switch fabric



Nrof cross-points: $p^2q + q^2p + p^2q = 2p^2q + q^2p$

Recursive construction of a switch fabric -2



Nrof Cross-points:

$$p(2p-1)q + q^2(2p-1) + (2p-1)pq = 2p(2p-1)q + q^2(2p-1)$$

Recursive construction of a switch fabric

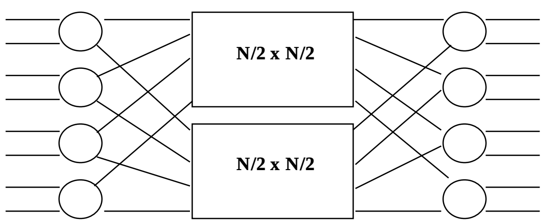
- ✔ **In recursive construction of a fabric, CLOS network is used to break the total switch into first order planes, then the planes are again broken down into smaller planes recursively. The same breakdown principle is applied each time.**
- ✔ **The planes of a Strict sense non-blocking Clos network are constructed out of strict sense Clos networks, etc...**
- ✔ **If strict sense planes or switch blocks are used, the result may be only rearrangeably non-blocking (see Clos theorem)**
- ✔ **We will show that if rearrangeably non-blocking planes or switch blocks are used, the result may be strict sense non-blocking under certain conditions!**

Problematics of construction

- ✔ **Ideal solution:**
  - **Minimum nrof cross-points**
  - **Low complexity**
  - **Simple to construct, etc … Goals may be conflicting.**
- ✔ **NxN switch fabric, how to choose P and Q ???**
- ✔ **To get the minimum nrof planes in the first and third stages, we can assume a small e.g. P = 2 --> Q = N/P.**
- ✔ **Then in the middle stage, the planes are still large Q x Q. We have solved only a minor part of the problem.**
- ✔ **Factor Q relates to the multiplexing factor (nrof time slots on inputs) --> we need to continue recursion until the speed of signals is low enough for our components. This may be useful for broadband!**

Case of 2x2 switch blocks

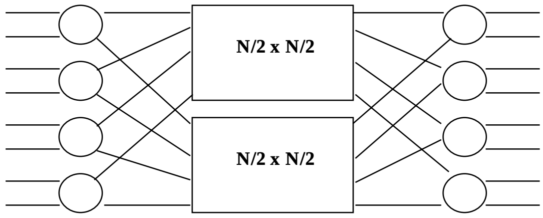
- ✔ **If the nrof inputs is a power of 2: ($N = 2^n$), the fabric can be constructed by assuming P=2 ja Q=N/2**



NB: Note the order of lines is based on the Clos principle!
The nrof first and last stage 2x2 switches is arbitrarily chosen in this example.

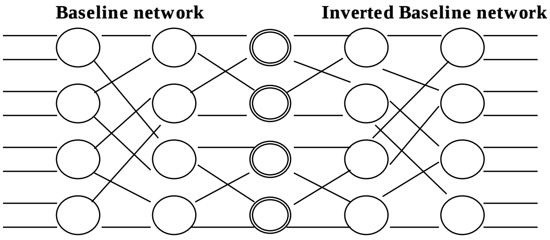
Example N=8

1)



2)

BENES network



Benes network

- ✔ **Benes network is recursively constructed of 2x2 switches and it is rearrengeably non-blocking based on the Clos theorem.**
- ✔ **1st half network is called the baseline network**
- ✔ **2nd half network is a mirror image of the first and we call it the inverted baseline network**
- ✔ **Benes network has ($2\log_2 N-1$) stages**
- ✔ **Nrof cross-points is**

$$4(N/2)(2\log_2 N-1) = 4N\log_2 N-2N \sim 4N\log_2 N$$

Constructing a strict sense non-blocking switch fabric using recursion

- ✔ **A strict sense non-blocking network can be similarly constructed, but the size grows much faster as function of nrof inputs.**
- ✔ **E.g. NxN -fabric can be constructed using $\sqrt{N} x \sqrt{N}$ switch blocks.**

- ✔ **Let us take $N = 2^n$ ja $n = 2^l$. Then the planes in the fabric are**
  - **1st stage: ($2^{n/2}$ x $2^{n/2}$) switch blocks, their nr = $2^{n/2}$**
  - **2nd stage: ($2^{n/2}$ x $2^{n/2}$) switch blocks, their nr = ($2x2^{n/2}$ -1)**
  - **3rd stage: ($2^{n/2}$ x $2^{n/2}$) switch blocks, their nr = $2^{n/2}$**

Continued

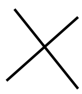
- ✔ **To be a bit more accurate, and ignoring some details.**
  - **1st stage: ($2^{n/2}$ x $2^{n/2+1}$) switch blocks, their nr = $2^{n/2}$**
  - **2nd stage: ($2^{n/2}$ x $2^{n/2}$) switch blocks, their nr = $2x2^{n/2}$ (ignore -1)**
  - **3rd stage: ($2^{n/2+1}$ x $2^{n/2}$) switch blocks, their nr = $2^{n/2}$**
  - **To simplify(ignore non-symmetry): ($2^{n/2}$ x $2^{n/2+1}$) = 2($2^{n/2}$ x $2^{n/2}$)**
  - **only ($2^{n/2}$ x $2^{n/2}$) switches, their total nr = $6(2^{n/2})$**
- ✔ **Recursive formula for cross-point complexity**

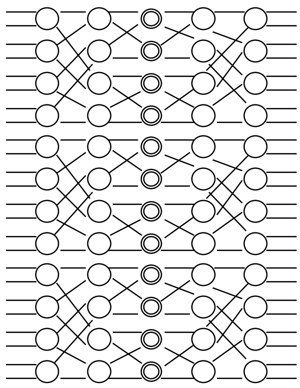
$$F(2^n)= 6(2^{n/2})F(2^{n/2}) = 6^l(2^{n/2+n/4+n/8+\ldots+1})F(2^1)$$
$$\sim N (\log_2 N)^{2,58} F(2) = 4N(\log_2 N)^{2,58}$$

- ✔ **Cmp to BENES: strict sense non-blocking fabric has more cross-points, the difference is the power!**

Cantor network is a way to construct a strict sense non-blocking fabric with a smaller nrof cross-points

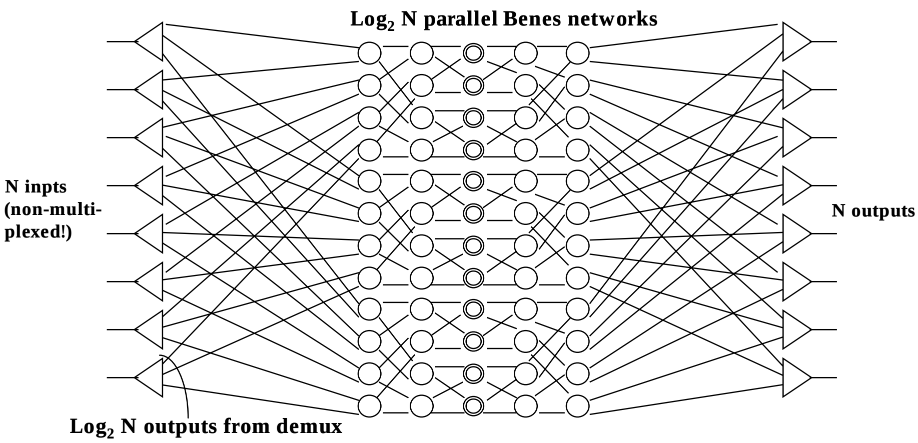
- ✔ **BENES network is taken as the building block here we see three of them**



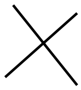


Cantor network

- ✔ **Cantor network is a way to construct a strict sense fabric with a smaller nrof cross-points.**



Properties of the Cantor network

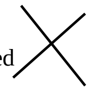


- **Nrof cross-points = $4 \times N\log_2 N \times \log_2 N = 4N(\log_2 N)^2$** if the muxes and demuxes are ignored.

Theorem: Cantor network is strict sense non-blocking!
Proof:
Mark nrof parallel Benes networks with *m* and the number of the stage in the Benes network with *k* and
A(*k*) - nrof reachable 2x2 switches without rearrangements in stage *k* starting from one input of the Cantor network.
$A(1) = m$.
$A(2) = 2A(1) - 1$.
$A(3) = 2A(2) - 2$.
$A(k) = 2A(k-1) - 2^{k-2} = 2^2A(k-2) - 2\times 2^{k-2} = 2^{k-1}A(1) - (k-1)2^{k-2}$
$A(\log N) = 2^{\log N - 1}m - (\log N - 1)2^{\log N - 2}$

$$= \frac{1}{2}Nm - \frac{1}{4}(\log N - 1)N$$

Cantor todistus jatkuu



$$2 \times \left(\frac{1}{2}Nm - \frac{1}{4}(\log N - 1)N\right) > \frac{Nm}{2}$$

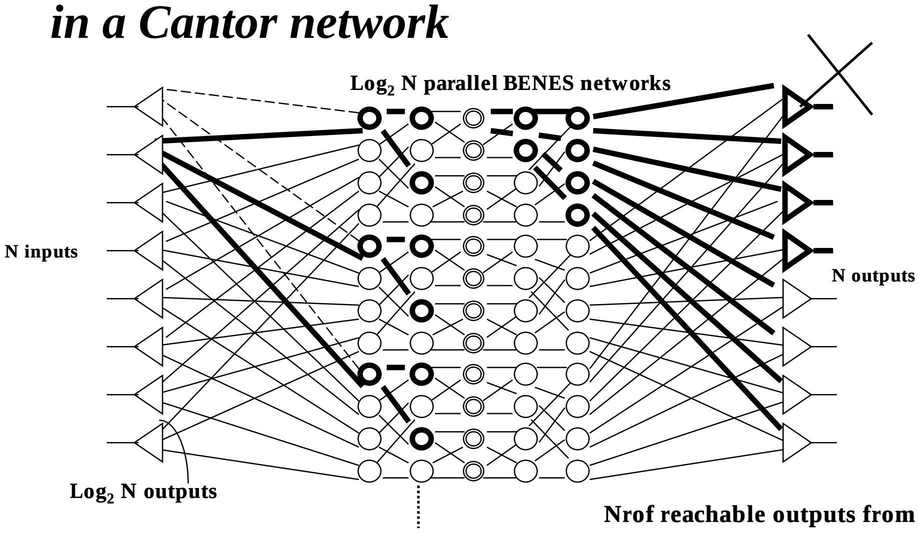
Because the inverted baseline network is exactly the same as the baseline network. The non-equality says that we still can find one extra 2x2 switch for our thruconnection

$\Rightarrow \boldsymbol{m} > \log N - 1$.

I.e. if the nrof parallel Benes networks is logN, Cantor network is strict sense non-blocking.

NB. *Strict sense non-blocking* Cantor network is constructed using logN only *rearrangeably non-blocking* Benes networks as building blocks!

Visualization of reachable 2x2 switches in a Cantor network



A(2) = 2A(1) – 1 (with no rearrangements)

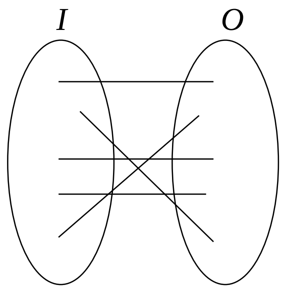
Nrof reachable outputs from a 2x2 switch in an inverted baseline network

Numeric example of a Cantor network

N = 32 x 2048 = 2^{16} ≈ 64 000
m = logN = 16
Nrof outputs in Demultiplexers = 16
Nrof inputs in Multiplexers = 16
Nrof Muxes = 64 000
Nrof Demuxes = 64 000
Nrof Benes networks = 16 kappaletta

Nrof stages in Benes networks = 2logN - 1 = 2x16-1 = 31
Nrof 2x2 switches in Benes networks= $N\log_2 N = 2^{16} * 32$ ≈ 2M in each one of them.

Total nrof thruconnections in a Fabric

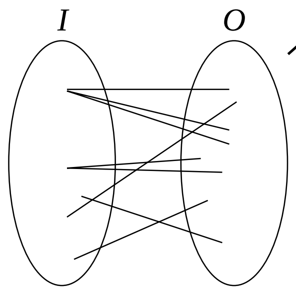


Point-to-point

$C = \{(i,o) | i \in I, o \in O\}$

$(i,o) \in C$ ja $(i,o') \in C \Rightarrow o = o'$

$(i,o) \in C$ ja $(i',o) \in C \Rightarrow i = i'$



One to many

$C = \{(i,n_i) | i \in I, n_i \subset O\}$

C - is a logical mapping from inputs to outputs.

C - a description of a connection of state of the Fabric

A complexity measure of a Fabric

- ✔ **A Complexity measure of a Fabric is the nrof cross-points in the Fabric.**
- ✔ **G is the size of the set of sets C{i,o}.**
- ✔ **$\zeta(G)$ is $\log_2(G)$ i.e. the logarithm of the nrof different states of of the Fabric (~ nrof cross-point, since each cross-point has two possible states)**
- ✔ **$\zeta(G)$ is used to approximate the complexity of a Fabric because NxN -matrix contains the largest nrof cross-points among all interesting architectures of a given size.**
  - **If a switch matrix has R cross-points, it has the the max of 2^R different states (each cp can be either open or closed), It follows that $\zeta <= R$ (taking into account that not all states are feasible).**

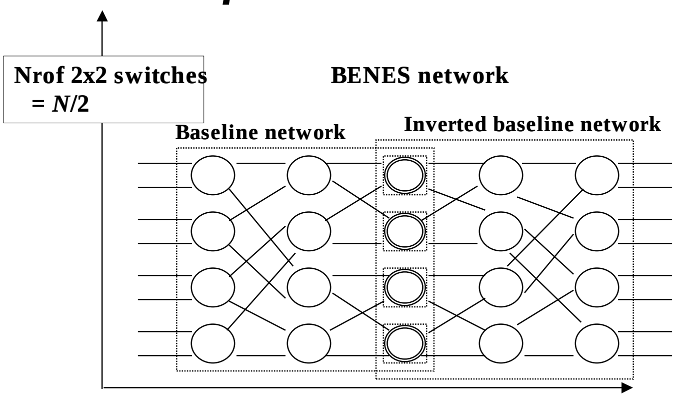
Lower bound of Complexity

- ✔ Assume that a Fabric is NxN and it provides full connectivity.
- ✔ G = N!
- ✔ $\zeta = \log_2 (N!) \sim N\log_2(N) - 1{,}44N + ½\log_2(N)$
- ✔ In the Benes network

$$\zeta \sim (N/2)(2\log_2 N-1) = N\log_2(N) - ½N$$

which is close to the lower bound.

Growth of the Benes network



Nrof stages = $2 \log_2 N - 1$

Nrof cross-points is nrof-stages × nrof-switches-in-a-stage =

$$4 \times N/2 \times (2 \log_2 N - 1) \approx 4N \log_2 N$$