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Simple teletraffic model

- **Customers arrive** at rate $\lambda$ (customers per time unit)
  - $1/\lambda = \text{average inter-arrival time}$
- Customers are **served** by $n$ parallel **servers**
- When busy, a server serves at rate $\mu$ (customers per time unit)
  - $1/\mu = \text{average service time of a customer}$
- There are $n + m$ **customer places** in the system
  - at least $n$ **service places** and at most $m$ **waiting places**
- It is assumed that blocked customers (arriving in a full system) are lost
7. Loss systems

**Infinite system**

- Infinite number of servers \((n = \infty)\), no waiting places \((m = 0)\)
  - No customers are lost or even have to wait before getting served
- Sometimes,
  - this hypothetical model can be used to get some approximate results for a real system (with finite system capacity)
- Always,
  - it gives bounds for the performance of a real system (with finite system capacity)
  - it is much easier to analyze than the corresponding finite capacity models
Pure loss system

- Finite number of servers \( n < \infty \), \( n \) service places, no waiting places \( m = 0 \)
  - If the system is full (with all \( n \) servers occupied) when a customer arrives, it is not served at all but lost
  - Some customers may be lost
- From the customer’s point of view, it is interesting to know e.g.
  - What is the probability that the system is full when it arrives?
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Poisson model ($M/M/\infty$)

- **Definition:** Poisson model is the following simple teletraffic model:
  - Infinite number of independent customers ($k = \infty$)
  - Interarrival times are IID and exponentially distributed with mean $1/\lambda$
    - so, customers arrive according to a Poisson process with intensity $\lambda$
  - **Infinite** number of servers ($n = \infty$)
  - Service times are IID and exponentially distributed with mean $1/\mu$
  - No waiting places ($m = 0$)

- **Poisson model:**
  - Using Kendall’s notation, this is an $M/M/\infty$ queue
  - Infinite system, and, thus, **lossless**

- **Notation:**
  - $a = \lambda/\mu = $ traffic intensity
7. Loss systems

State transition diagram

- Let $X(t)$ denote the number of customers in the system at time $t$
  - Assume that $X(t) = i$ at some time $t$, and consider what happens during a short time interval $(t, t+h]$:
    - with prob. $\lambda h + o(h)$, a new customer arrives (state transition $i \rightarrow i+1$)
    - if $i > 0$, then, with prob. $i\mu h + o(h)$, a customer leaves the system (state transition $i \rightarrow i-1$)
- Process $X(t)$ is clearly a Markov process with state transition diagram

- Note that process $X(t)$ is an irreducible birth-death process with an infinite state space $S = \{0,1,2,...\}$
7. Loss systems

Equilibrium distribution (1)

- **Local balance equations (LBE):**

\[
\pi_i \lambda = \pi_{i+1} (i + 1) \mu \quad \text{(LBE)}
\]

\[
\Rightarrow \quad \pi_{i+1} = \frac{\lambda}{(i+1)\mu} \pi_i = \frac{a}{i+1} \pi_i
\]

\[
\Rightarrow \quad \pi_i = \frac{a^i}{i!} \pi_0, \quad i = 0, 1, 2, \ldots
\]

- **Normalizing condition (N):**

\[
\sum_{i=0}^{\infty} \pi_i = \pi_0 \sum_{i=0}^{\infty} \frac{a^i}{i!} = 1 \quad \text{(N)}
\]

\[
\Rightarrow \quad \pi_0 = \left( \sum_{i=0}^{\infty} \frac{a^i}{i!} \right)^{-1} = \left( e^a \right)^{-1} = e^{-a}
\]

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7. Loss systems

Equilibrium distribution (2)

• Thus, the equilibrium distribution is a Poisson distribution:

\[ X \sim \text{Poisson}(a) \]

\[ P\{X = i\} = \pi_i = \frac{a^i}{i!} e^{-a}, \quad i = 0,1,2,\ldots \]

\[ E[X] = a, \quad D^2[X] = a \]

• **Remark:** Insensitivity with respect to service time distribution
  - The result is **insensitive** to the service time distribution, that is: it is valid for any service time distribution with mean \( 1/\mu \)
  - So, instead of the M/M/∞ model, we can consider, as well, the more general M/G/∞ model
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Application to flow level modelling of streaming data traffic

- Poisson model may be applied to flow level modelling of streaming data traffic
  - customer = UDP flow with constant bit rate (CBR)
  - $\lambda$ = flow arrival rate (flows per time unit)
  - $h = 1/\mu = $ average flow duration (time units)
  - $a = \lambda/\mu = $ traffic intensity
  - $r = $ bit rate of a flow (data units per time unit)
  - $N = nr$ of active flows obeying Poisson($a$) distribution

- When the total transmission rate $Nr$ exceeds the link capacity $C = nr$, bits are lost
  - **loss ratio** $P_{\text{loss}}$ gives the ratio between the traffic lost and the traffic offered:

$$P_{\text{loss}} = \frac{E[(Nr-C)^+]}{E[Nr]} = \frac{E[(N-n)^+]}{E[N]} = \frac{1}{a} \sum_{i=n+1}^{\infty} \frac{(i-n)a^i}{i!} e^{-a}$$
7. Loss systems

**Multiplexing gain**

- We determine traffic intensity $a$ so that loss ratio $p_{\text{loss}} < 1\%$
- **Multiplexing gain** is described by the traffic intensity per capacity unit, $a/n$, as a function of capacity $n$
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**Erlang model (M/M/n/n)**

- **Definition:** Erlang model is the following simple teletraffic model:
  - Infinite number of independent customers \( k = \infty \)
  - Interarrival times are IID and exponentially distributed with mean \( 1/\lambda \)
    - so, customers arrive according to a Poisson process with intensity \( \lambda \)
  - **Finite** number of servers \( n < \infty \)
  - Service times are IID and exponentially distributed with mean \( 1/\mu \)
  - No waiting places \( m = 0 \)

- **Erlang model:**
  - Using Kendall’s notation, this is an M/M/n/n queue
  - Pure loss system, and, thus, **lossy**

- **Notation:**
  - \( a = \lambda/\mu = \text{traffic intensity} \)
7. Loss systems

State transition diagram

- Let $X(t)$ denote the number of customers in the system at time $t$
  - Assume that $X(t) = i$ at some time $t$, and consider what happens during a short time interval $(t, t+h]$:
    - with prob. $\lambda h + o(h)$, a new customer arrives (state transition $i \rightarrow i+1$)
    - with prob. $i\mu h + o(h)$, a customer leaves the system (state transition $i \rightarrow i-1$)
- Process $X(t)$ is clearly a Markov process with state transition diagram
- Note that process $X(t)$ is an irreducible birth-death process with a finite state space $S = \{0,1,2,\ldots,n\}$
7. Loss systems

Equilibrium distribution (1)

- Local balance equations (LBE):

\[ \pi_i \lambda = \pi_{i+1} (i+1) \mu \]  

\[ \Rightarrow \pi_{i+1} = \frac{\lambda}{(i+1) \mu} \pi_i = \frac{a_i}{i+1} \pi_i \]

\[ \Rightarrow \pi_i = \frac{a_i}{i!} \pi_0, \quad i = 0, 1, \ldots, n \]

- Normalizing condition (N):

\[ \sum_{i=0}^{n} \pi_i = \pi_0 \sum_{i=0}^{n} \frac{a_i}{i!} = 1 \]

\[ \Rightarrow \pi_0 = \left( \sum_{i=0}^{n} \frac{a_i}{i!} \right)^{-1} \]
7. Loss systems

**Equilibrium distribution (2)**

- Thus, the equilibrium distribution is a **truncated Poisson distribution**:

\[
P\{X = i\} = \pi_i = \frac{a^i}{i!}, \quad i = 0, 1, \ldots, n
\]

- **Remark**: Insensitivity with respect to the service time distribution
  - The result is **insensitive** to the service time distribution, that is: it is valid for **any** service time distribution with mean 1/\(\mu\)
  - So, instead of the M/M/n/n model, we can consider, as well, the more general M/G/n/n model
7. Loss systems

**Time blocking**

- **Time blocking** $B_t = \text{probability that all } n \text{ servers are occupied at an arbitrary time} = \text{the fraction of time that all } n \text{ servers are occupied}

- For a stationary Markov process, this equals the probability $\pi_n$ of the equilibrium distribution $\pi$. Thus,

$$B_t := P\{X = n\} = \pi_n = \frac{a^n}{n!} \sum_{j=0}^{n} \frac{a^j}{j!}$$
Call blocking

- **Call blocking** $B_c = \text{probability that an arriving customer finds all } n \text{ servers occupied} = \text{the fraction of arriving customers that are lost}

- However, due to Poisson arrivals and PASTA property, the probability that an arriving customer finds all $n$ servers occupied equals the probability that all $n$ servers are occupied at an arbitrary time,

- In other words, call blocking $B_c$ equals time blocking $B_t$:

$$B_c = B_t = \frac{a^n}{n!} \sum_{j=0}^{n} \frac{a^j}{j!}$$

- This is **Erlang’s blocking formula**
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• Application to telephone traffic modelling in access network
Application to telephone traffic modelling in trunk network

- Erlang model may be applied to modelling of telephone traffic in trunk network where the number of potential users of a link is large
  - customer = call
  - \( \lambda \) = call arrival rate (calls per time unit)
  - \( h = \frac{1}{\mu} \) = average call holding time (time units)
  - \( a = \frac{\lambda}{\mu} \) = traffic intensity
  - \( n \) = link capacity (channels)

- A call is lost if all \( n \) channels are occupied when the call arrives
  - call blocking \( B_c \) gives the probability of such an event

\[
B_c = \frac{\frac{a^n}{n!}}{\sum_{j=0}^{n-1} \frac{a^j}{j!}}
\]
7. Loss systems

Multiplexing gain

- We determine traffic intensity $a$ so that call blocking $B_c < 1\%$
- **Multiplexing gain** is described by the traffic intensity per capacity unit, $a/n$, as a function of capacity $n$
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Binomial model \((M/M/k/k/k)\)

- **Definition:** Binomial model is the following (simple) teletraffic model:
  - **Finite** number of independent customers \((k < \infty)\)
  - **on-off type** customers (alternating between idleness and activity)
    - Idle times are IID and exponentially distributed with mean \(1/\nu\)
    - As many servers as customers \((n = k)\)
    - Service times are IID and exponentially distributed with mean \(1/\mu\)
    - No waiting places \((m = 0)\)
  - **Binomial model:**
    - Using Kendall’s notation, this is an \(M/M/k/k/k\) queue
    - Although a finite system, this is clearly **lossless**
- **On-off type customer:**

![Diagram of on-off type customer](image)
On-off type customer (1)

- Let $X_j(t)$ denote the state of customer $j$ ($j = 1, 2, \ldots, k$) at time $t$
  - State 0 = idle, state 1 = active = in service
  - Consider what happens during a short time interval $(t, t+h]$:
    - if $X_j(t) = 0$, then, with prob. $\nu h + o(h)$, the customer becomes active (state transition $0 \rightarrow 1$)
    - if $X_j(t) = 1$, then, with prob. $\mu h + o(h)$, the customer becomes idle (state transition $1 \rightarrow 0$)
- Process $X_j(t)$ is clearly a Markov process with state transition diagram

\[ \begin{array}{c}
0 \qquad \nu \qquad 1 \\
\mu 
\end{array} \]

- Note that process $X_j(t)$ is an irreducible birth-death process with a finite state space $S = \{0, 1\}$
7. Loss systems

On-off type customer (2)

- Local balance equations (LBE):
  \[
  \pi_0^{(j)} \nu = \pi_1^{(j)} \mu \quad \Rightarrow \quad \pi_1^{(j)} = \frac{\nu}{\mu} \pi_0^{(j)}
  \]

- Normalizing condition (N):
  \[
  \pi_0^{(j)} + \pi_1^{(j)} = \pi_0^{(j)} \left(1 + \frac{\nu}{\mu}\right) = 1 \quad \Rightarrow \quad \pi_0^{(j)} = \frac{\mu}{\nu + \mu}, \quad \pi_1^{(j)} = \frac{\nu}{\nu + \mu}
  \]

- So, the equilibrium distribution of a single customer is the Bernoulli distribution with success probability \( \nu/(\nu+\mu) \)
  - offered traffic is \( \nu/(\nu+\mu) \)

- From this, we could deduce that the equilibrium distribution of the state of the whole system (that is: the number of active customers) is the binomial distribution \( \text{Bin}(k, \nu/(\nu+\mu)) \)
7. Loss systems

State transition diagram

- Let \( X(t) \) denote the number of active customers
  - Assume that \( X(t) = i \) at some time \( t \), and consider what happens during a short time interval \( (t, t+h] \):
    - if \( i < k \), then, with prob. \( (k-i)\nu h + o(h) \), an idle customer becomes active (state transition \( i \rightarrow i+1 \))
    - if \( i > 0 \), then, with prob. \( i\mu h + o(h) \), an active customer becomes idle (state transition \( i \rightarrow i-1 \))
- Process \( X(t) \) is clearly a Markov process with state transition diagram

\[
\begin{array}{cccccc}
 0 & \overset{k\nu}{\underset{\mu}{\longrightarrow}} & 1 & \overset{(k-1)\nu}{\underset{2\mu}{\longrightarrow}} & \cdots & \overset{2\nu}{\underset{(k-1)\mu}{\longrightarrow}} & k-1 & \overset{\nu}{\underset{k\mu}{\longrightarrow}} & k
\end{array}
\]

- Note that process \( X(t) \) is an irreducible birth-death process with a finite state space \( S = \{0,1,\ldots,k\} \)
Equilibrium distribution (1)

- Local balance equations (LBE):

\[ \pi_i (k - i) \nu = \pi_{i+1} (i + 1) \mu \]  

\Rightarrow \quad \pi_{i+1} = \frac{(k-i)\nu}{(i+1)\mu} \pi_i

\Rightarrow \quad \pi_i = \frac{k!}{i!(k-i)!} \left( \frac{\nu}{\mu} \right)^i \pi_0 = \binom{k}{i} \left( \frac{\nu}{\mu} \right)^i \pi_0, \quad i = 0, 1, \ldots, k

- Normalizing condition (N):

\[ \sum_{i=0}^{k} \pi_i = \pi_0 \sum_{i=0}^{k} \binom{k}{i} \left( \frac{\nu}{\mu} \right)^i = 1 \]  

\Rightarrow \quad \pi_0 = \left( \sum_{i=0}^{k} \binom{k}{i} \left( \frac{\nu}{\mu} \right)^i \right)^{-1} = (1 + \frac{\nu}{\mu})^{-k} = \left( \frac{\mu}{\nu + \mu} \right)^k

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Equilibrium distribution (2)

- Thus, the equilibrium distribution is a **binomial distribution**:

\[
X \sim \text{Bin}(k, \frac{\nu}{\nu+\mu})
\]

\[
P\{X = i\} = \pi_i = \binom{k}{i} \left(\frac{\nu}{\nu+\mu}\right)^i \left(\frac{\mu}{\nu+\mu}\right)^{k-i}, \quad i = 0,1,\ldots,k
\]

\[
E[X] = \frac{k\nu}{\nu+\mu}, \quad D^2[X] = k \cdot \frac{\nu}{\nu+\mu} \cdot \frac{\mu}{\nu+\mu} = \frac{k\nu\mu}{(\nu+\mu)^2}
\]

- **Remark**: Insensitivity w.r.t. service time and idle time distribution
  - The result is **insensitive both** to the service **and** the idle time distribution, that is: it is valid for **any** service time distribution with mean $1/\mu$ and **any** idle time distribution with mean $1/\nu$
  - So, instead of the $M/M/k/k/k$ model, we can consider, as well, the more general $G/G/k/k/k$ model
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Engset model \((M/M/n/n/k)\)

- **Definition**: Engset model is the following (simple) teletraffic model:
  - **Finite** number of independent customers \((k < \infty)\)
    - **on-off type** customers (alternating between idleness and activity)
  - Idle times are IID and exponentially distributed with mean \(1/\nu\)
  - **Less servers** than customers \((n < k)\)
  - Service times are IID and exponentially distributed with mean \(1/\mu\)
  - No waiting places \((m = 0)\)

- **Engset model**:
  - Using Kendall’s notation, this is an \(M/M/n/n/k\) queue
  - This is a pure loss system, and, thus, **lossy**

- **On-off type customer**:
  - Note: If the system is full when an idle cust. tries to become an active cust., a new idle period starts.
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**State transition diagram**

- Let $X(t)$ denote the number of active customers
  - Assume that $X(t) = i$ at some time $t$, and consider what happens during a short time interval $(t, t+h]$:
    - if $i < n$, then, with prob. $(k-i)\nu h + o(h)$, an idle customer becomes active (state transition $i \rightarrow i+1$)
    - if $i > 0$, then, with prob. $i\mu h + o(h)$, an active customer becomes idle (state transition $i \rightarrow i-1$)
  - Process $X(t)$ is clearly a Markov process with state transition diagram

![State transition diagram](diagram)

- Note that process $X(t)$ is an irreducible birth-death process with a finite state space $S = \{0,1,\ldots,n\}$
7. Loss systems

Equilibrium distribution (1)

- **Local balance equations (LBE):**

\[
\pi_i (k - i) \nu = \pi_{i+1} (i + 1) \mu \\
\Rightarrow \pi_{i+1} = \frac{(k - i) \nu}{(i + 1) \mu} \pi_i \\
\Rightarrow \pi_i = \frac{k!}{i!(k-i)!} \left(\frac{\nu}{\mu}\right)^i \pi_0 = \binom{k}{i} \left(\frac{\nu}{\mu}\right)^i \pi_0, \quad i = 0, 1, \ldots, n
\]

- **Normalizing condition (N):**

\[
\sum_{i=0}^{n} \pi_i = \pi_0 \sum_{i=0}^{n} \binom{k}{i} \left(\frac{\nu}{\mu}\right)^i = 1 \\
\Rightarrow \pi_0 = \left(\sum_{i=0}^{n} \binom{k}{i} \left(\frac{\nu}{\mu}\right)^i\right)^{-1}
\]
Equilibrium distribution (2)

- Thus, the equilibrium distribution is a **truncated binomial distribution**:

\[
P\{X = i\} = \pi_i = \frac{{\binom{k}{i}} \left(\frac{\nu}{\mu}\right)^i}{\sum_{j=0}^{n} \binom{k}{j} \left(\frac{\nu}{\mu}\right)^j} = \frac{{\binom{k}{i}} \left(\frac{\nu}{\nu+\mu}\right)^i \left(\frac{\mu}{\nu+\mu}\right)^{k-i}}{\sum_{j=0}^{n} \binom{k}{j} \left(\frac{\nu}{\nu+\mu}\right)^j \left(\frac{\mu}{\nu+\mu}\right)^{k-j}}, i = 0, \ldots, n
\]

- Offered traffic is \(k\nu/(\nu+\mu)\)

- **Remark**: Insensitivity w.r.t. service time and idle time distribution
  - The result is **insensitive both** to the service **and** the idle time distribution, that is: it is valid for **any** service time distribution with mean 1/\(\mu\) and **any** idle time distribution with mean 1/\(\nu\)
  - So, instead of the M/M/n/n/k model, we can consider, as well, the more general G/G/n/n/k model
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**Time blocking**

- **Time blocking** \( B_t \) = probability that all \( n \) servers are occupied at an arbitrary time = the fraction of time that all \( n \) servers are occupied.
- For a stationary Markov process, this equals the probability \( \pi_n \) of the equilibrium distribution \( \pi \). Thus,

\[
B_t := P\{X = n\} = \pi_n = \frac{\binom{k}{n} \left( \frac{v}{\mu} \right)^n}{\sum_{j=0}^{n} \binom{k}{j} \left( \frac{v}{\mu} \right)^j}
\]
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Call blocking (1)

- **Call blocking** $B_c = \text{probability that an arriving customer finds all } n \text{ servers occupied} = \text{the fraction of arriving customers that are lost}
  - In the Engset model, however, the “arrivals” do **not** follow a Poisson process. Thus, we cannot utilize the PASTA property any more.
  - In fact, the distribution of the state that an “arriving” customer sees differs from the equilibrium distribution. Thus, call blocking $B_c$ does **not** equal time blocking $B_t$ in the Engset model.
Call blocking (2)

- Let $\pi^*_i$ denote the probability that there are $i$ active customers when an idle customer becomes active (which is called an “arrival”)

- Consider a long time interval $(0, T)$:
  - During this interval, the average time spent in state $i$ is $\pi_i T$
  - During this time, the average number of “arriving” customers (who all see the system to be in state $i$) is $(k-i)\nu \cdot \pi_i T$
  - During the whole interval, the average number of “arriving” customers is $\sum_j (k-j)\nu \cdot \pi_j T$

- Thus,

\[
\pi^*_i = \frac{(k-i)\nu \cdot \pi_i T}{\sum_{j=0}^{n} (k-j)\nu \cdot \pi_j T} = \frac{(k-i)\cdot \pi_i}{\sum_{j=0}^{n} (k-j) \cdot \pi_j}, \quad i = 0, 1, \ldots, n
\]
Call blocking (3)

- It can be shown (exercise!) that

\[ \pi_i^* = \frac{\binom{k-1}{i}(\nu)^i(\frac{\nu}{\mu})^i}{\sum_{j=0}^{n} \binom{k-1}{j}(\nu)^j(\frac{\nu}{\mu})^j}, \quad i = 0, 1, \ldots, n \]

- If we write explicitly the dependence of these probabilities on the total number of customers, we get the following result:

\[ \pi_i^*(k) = \pi_i(k-1), \quad i = 0, 1, \ldots, n \]

- In other words, an “arriving” customer sees such a system where there is one customer less (itself!) in equilibrium.
Call blocking (4)

• By choosing \( i = n \), we get the following formula for the call blocking probability:

\[
B_c(k) = \pi_n \ast (k) = \pi_n (k - 1) = B_t(k - 1)
\]

• Thus, for the Engset model, the call blocking in a system with \( k \) customers equals the time blocking in a system with \( k - 1 \) customers:

\[
B_c(k) = B_t(k - 1) = \frac{\binom{k-1}{n}\left(\frac{\nu}{\mu}\right)^n}{\sum_{j=0}^{n} \binom{k-1}{j}\left(\frac{\nu}{\mu}\right)^j}
\]

• This is Engset’s blocking formula
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Application to telephone traffic modelling in access network

- Engset model may be applied to modelling of telephone traffic in access network where the nr of potential users of a link is moderate
  - customer = call
  - $\nu = \text{call arrival rate per idle user (calls per time unit)}$
  - $1/\mu = \text{average call holding time (time units)}$
  - $k = \text{number of potential users}$
  - $n = \text{link capacity (channels)}$

- A call is lost if all $n$ channels are occupied when the call arrives
  - **call blocking** $B_c$ gives the probability of such an event

\[
B_c = \frac{\left(\frac{k-1}{n}\right)\left(\frac{\nu}{\mu}\right)^n}{\sum_{j=0}^{n} \left(\frac{k-1}{j}\right)\left(\frac{\nu}{\mu}\right)^j}
\]
7. Loss systems

**Multiplexing gain**

- We assume that an access link is loaded by $k = 100$ potential users.
- We determine traffic intensity $k\nu/(\nu+\mu)$ so that call blocking $B_c < 1\%$.
- **Multiplexing gain** is described by the traffic intensity per capacity unit, $k\nu/(n(\nu+\mu))$, as a function of capacity $n$. 

![Graph showing multiplexing gain as a function of capacity](image)

- The y-axis represents the normalized traffic $k\nu/(n(\nu+\mu))$.
- The x-axis represents the capacity $n$. 

The graph illustrates how the normalized traffic increases as the capacity $n$ increases.
7. Loss systems

THE END