Exercise 3
5.2.2008

1. A link in a packet switched network carries on average 1 packet/ms. Assume that packets arrive according to a Poisson process. Each packet is a data packet with probability 0.9 and an acknowledgement (ACK) with probability 0.1, independently of the other packets. Consider a random time interval of length 1 ms.

   a) What is the probability that exactly two data packets but no ACKs arrive during the interval?

   b) Assume now that two packet arrivals have been observed during the interval. What is the probability that they both are data packets?

2. A Markov process is defined in the state space $\{0, 1, 2, 3\}$ with the state transitions rates q_{ij} collected in the transition matrix $Q = (q_{ij} \mid i, j = 0, 1, 2, 3)$, where $q_{ii} = -q_i$ for all i, as follows:

$$Q = \begin{pmatrix} -5 & 2 & 0 & 3 \\ 1 & -7 & 6 & 0 \\ 0 & 2 & -4 & 2 \\ 1 & 0 & 4 & -5 \end{pmatrix}$$

   (a) Draw the state transition diagram of $X(t)$.

   (b) Derive the equilibrium distribution of $X(t)$.

   (c) Is the process reversible? (In other words, are the local balance equations (LBE) satisfied?)

3. A Markov process is defined in the state space $\{0, 1, 2, 3\}$ with the state transitions rates q_{ij} collected in the transition matrix $Q = (q_{ij} \mid i, j = 0, 1, 2, 3)$, where $q_{ii} = -q_i$ for all i, as follows:

$$Q = \begin{pmatrix} -2 & 2 & 0 & 0 \\ 1 & -5 & 4 & 0 \\ 0 & 2 & -8 & 6 \\ 0 & 0 & 3 & -3 \end{pmatrix}$$

   (a) Draw the state transition diagram of $X(t)$.

   (b) Derive the equilibrium distribution of $X(t)$.

   (c) Is the process reversible? (In other words, are the local balance equations (LBE) satisfied?)