TKK HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Communications and Networking
S-38.1145 Introduction to Teletraffic Theory, Spring 2008

Exercise 2
29.1.2008

1. Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed (i.i.d.) random variables, for which $X_i \sim \text{Exp}(\mu)$. The cumulative distribution function of a continuous random variable T is given by

$$P\{T \leq t\} = P\{\min(X_1, \ldots, X_n) \leq t\}, \quad t \geq 0$$

   (a) Which (known) continuous distribution do we have?

   (b) Determine expectation $E[T]$ and variance $D^2[T]$.

2. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables, for which $X_i \sim \text{Exp}(1)$. The cumulative distribution function of a discrete random variable N is given by

$$P\{N \leq n\} = P\{\max(X_1, \ldots, X_{n+1}) > 1\}, \quad n = 0, 1, \ldots$$

   (a) Which (known) discrete distribution do we have?

   (b) Determine expectation $E[N]$ and variance $D^2[N]$.

3. Let X be a continuous nonnegative random variable with cumulative distribution function $F(x) = P\{X \leq x\}$. Furthermore, let $c > 0$ be a constant.

   (a) Derive the following formula for the expectation of the random variable $\min\{X, c\}$:

$$E[\min\{X, c\}] = \int_0^c (1 - F(x))\, dx.$$

   (b) Based on the previous result, give a formula for $E[X]$.