D10/1 Consider the following simple circuit switched trunk network. There are three nodes connected in a tandem by two links: a — b — c. Each link has capacity of 2 channels. In addition, there are three traffic classes:

- Class 1 uses link a — b
- Class 2 uses link b — c
- Class 3 uses both link a — b and link b — c

(a) What is the state space of this system?

(b) Determine the blocking states for each class separately.

D10/2 Consider again the circuit switched trunk network defined in the previous problem. Assume that, for each class r, new calls arrive according to a Poisson process with rate λ_r. Let $\lambda_1 = 0$, $\lambda_2 = 1/3$, and $\lambda_3 = 2/3$ calls/min. Call holding times (for all classes) are assumed to be independently and identically distributed with mean $h = 3$ min. Compute the end-to-end blocking probabilities for each class using

(a) the exact formula,

(b) the approximative Product Bound method.

D10/3 Consider a connectionless packet switched trunk network with three nodes connected to each other as a triangle. Each node pair is connected with two one-way links (one in each direction) of capacity 1 Gbps. The following five routes are used in this network:

- Route 1: a → b
- Route 2: a → c → b
- Route 3: a → c
- Route 4: c → b
- Route 5: b → a

For each route, new packets arrive according to an independent Poisson process with intensities $\lambda(1) = 40$, $\lambda(2) = 60$, $\lambda(3) = \lambda(4) = \lambda(5) = 20$ packets/ms. The packet lengths are independent and exponentially distributed with mean 1250 bytes. Determine

(a) the traffic loads for each link j,

(b) the mean end-to-end delays for each route r.

D10/1 The system consists of two links with equal capacities $n_1 = n_2 = 2$. The state of the system is described by the vector $\mathbf{x} = (x_1, x_2, x_3)$ where x_r refers to the number of active connections on route r. The topology and the routes of this network are shown in Figure 1.

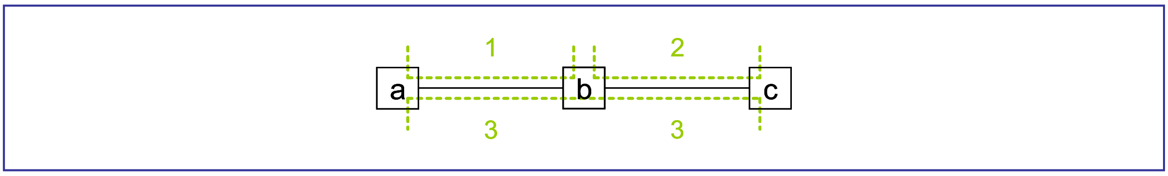

Figure 1: [D10/1] Topology and routes.

(a) The link capacities give the following bounds (L10/8) for x_r:

$$x_1 + x_3 \le n_1 = 2, \quad x_2 + x_3 \le n_2 = 2$$

Thus, the state space $\mathcal{S}$ is

$$\begin{aligned} \mathcal{S} \;=\; &\{(0,0,0),(1,0,0),(2,0,0),\\ &(0,1,0),(1,1,0),(2,1,0),\\ &(0,2,0),(1,2,0),(2,2,0),\\ &(0,0,1),(1,0,1),\\ &(0,1,1),(1,1,1),\\ &(0,0,2)\} \end{aligned}$$

The state space is illustrated in Figure 2.

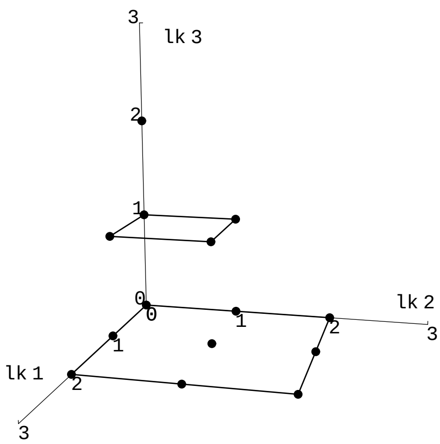

Figure 2: [D10/1] State space.

(b) 1° Class 1 uses only link 1 so that the set of non-blocking states $\mathcal{S}_1$ satisfies (L10/10)

$$x_1 + x_3 \le n_1 - 1 = 1, \quad x_2 + x_3 \le n_2 = 2$$

leading to

$$\mathcal{S}_1 = \{(0,0,0),(1,0,0),(0,1,0),(1,1,0),(0,2,0),(1,2,0),(0,0,1),(0,1,1)\}$$

Thus, the set of blocking states $\mathcal{S}_1^{\mathrm{B}} = \mathcal{S} \setminus \mathcal{S}_1$ is

$$\mathcal{S}_1^{\mathrm{B}} = \{(2,0,0),(2,1,0),(2,2,0),(1,0,1),(1,1,1),(0,0,2)\}$$

This set is depicted in Figure 3.

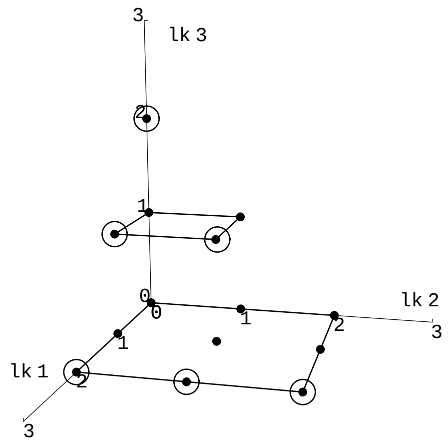

Figure 3: [D10/1] Set of blocking states for class 1 (circled).

2° Class 2 uses only link 2 so that the set of non-blocking states $\mathcal{S}_2$ satisfies (L10/10)

$$x_1 + x_3 \leq n_1 = 2, \quad x_2 + x_3 \leq n_2 - 1 = 1$$

leading to

$$\mathcal{S}_2 = \{(0,0,0),(1,0,0),(2,0,0),(0,1,0),(1,1,0),(2,1,0),(0,0,1),(1,0,1)\}$$

Thus, the set of blocking states $\mathcal{S}_2^{\mathrm{B}} = \mathcal{S} \setminus \mathcal{S}_2$ is

$$\mathcal{S}_2^{\mathrm{B}} = \{(0,2,0),(1,2,0),(2,2,0),(0,1,1),(1,1,1),(0,0,2)\}.$$

This set is depicted in Figure 4.

3° Class 3 uses both links so that the set of non-blocking states $\mathcal{S}_3$ satisfies (L10/10)

$$x_1 + x_3 \leq n_1 - 1 = 1, \quad x_2 + x_3 \leq n_2 - 1 = 1$$

leading to

$$\mathcal{S}_3 = \{(0,0,0),(1,0,0),(0,1,0),(1,1,0),(0,0,1)\}$$

Thus, the set of blocking states $\mathcal{S}_3^{\mathrm{B}} = \mathcal{S} \setminus \mathcal{S}_3$ is

$$\mathcal{S}_3^{\mathrm{B}} = \{(2,0,0),(2,1,0),(0,2,0),(1,2,0),(2,2,0),(1,0,1),(0,1,1),(1,1,1),(0,0,2)\}$$

This set is depicted in Figure 5.

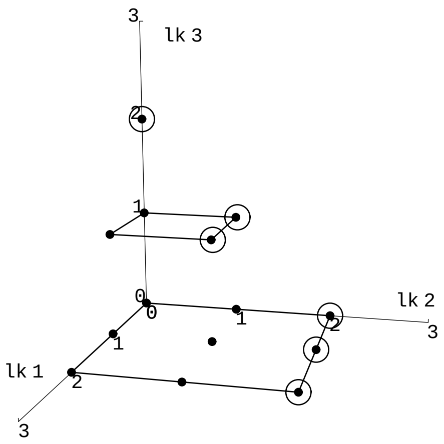

Figure 4: [D10/1] Set of blocking states for class 2 (circled).

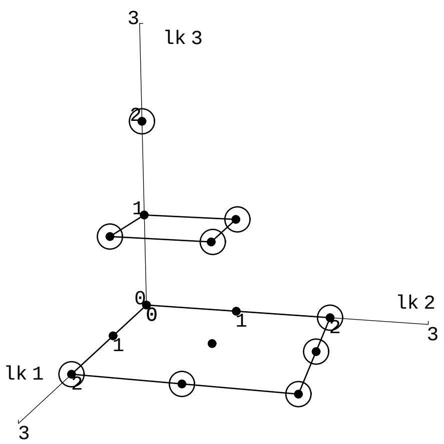

Figure 5: [D10/1] Set of blocking states for class 3 (circled).

D10/2 (a) The traffic intensities of the three classes are as follows (L10/12):

$$a_1 = \lambda_1 h = 0 \cdot 3 = 0 \text{ erl}, \quad a_2 = \lambda_2 h = \frac{1}{3} \cdot 3 = 1 \text{ erl}, \quad a_3 = \lambda_3 h = \frac{2}{3} \cdot 3 = 2 \text{ erl}$$

Since there is no traffic in class 1 ($a_1 = 0$), the state space shrinks down to

$$\mathcal{S} = \{(0,0,0), (0,1,0), (0,2,0), (0,0,1), (0,1,1), (0,0,2)\}$$

The equilibrium distribution is (L10/13)

$$\pi(\mathbf{x}) = \pi(0, x_2, x_3) = \pi(0,0,0) \frac{a_2^{x_2}}{x_2!} \frac{a_3^{x_3}}{x_3!} = \pi(0,0,0) \frac{1}{x_2!} \frac{2^{x_3}}{x_3!}$$

where

$$\pi(0,0,0) = \left(\sum_{\mathbf{x} \in \mathcal{S}} \frac{a_2^{x_2}}{x_2!} \frac{a_3^{x_3}}{x_3!} \right)^{-1} = \left(\sum_{\mathbf{x} \in \mathcal{S}} \frac{1}{x_2!} \frac{2^{x_3}}{x_3!} \right)^{-1}$$

Thus,

$$
\begin{aligned}
\pi(0,0,0) &= \frac{1}{1+1+\frac{1}{2}+2+2+2} = \frac{2}{17} = 0.118 \\
\pi(0,1,0) &= \pi(0,0,0)\frac{1}{1!}\frac{2^0}{0!} = \pi(0,0,0)\cdot 1 = \frac{2}{17} = 0.118 \\
\pi(0,2,0) &= \pi(0,0,0)\frac{1}{2!}\frac{2^0}{0!} = \pi(0,0,0)\cdot \frac{1}{2} = \frac{1}{17} = 0.059 \\
\pi(0,0,1) &= \pi(0,0,0)\frac{1}{0!}\frac{2^1}{1!} = \pi(0,0,0)\cdot 2 = \frac{4}{17} = 0.235 \\
\pi(0,1,1) &= \pi(0,0,0)\frac{1}{1!}\frac{2^1}{1!} = \pi(0,0,0)\cdot 2 = \frac{4}{17} = 0.235 \\
\pi(0,0,2) &= \pi(0,0,0)\frac{1}{0!}\frac{2^2}{2!} = \pi(0,0,0)\cdot 2 = \frac{4}{17} = 0.235
\end{aligned}
$$

Since class 1 remains empty, the set of blocking states $\mathcal{S}_1^{\mathrm{B}}$ is reduced to

$$\mathcal{S}_1^{\mathrm{B}} = \{(0,0,2)\}.$$

Thus, the (hypothetical) end-to-end call blocking probability for class 1 (L10/16) equals

$$B_1 = \pi(0,0,2) = \frac{4}{17} = 0.235$$

Correspondingly, the sets of blocking states $\mathcal{S}_2^{\mathrm{B}}$ and $\mathcal{S}_3^{\mathrm{B}}$ are reduced to

$$\mathcal{S}_2^{\mathrm{B}} = \mathcal{S}_3^{\mathrm{B}} = \{(0,2,0),(0,1,1),(0,0,2)\}$$

so that the end-to-end call blocking probabilities for classes 1 and 2 become

$$B_2 = B_3 = \pi(0,2,0)+\pi(0,1,1)+\pi(0,0,2) = \frac{1+4+4}{17} = \frac{9}{17} = 0.53$$

In fact, since $\mathcal{S}_2^{\mathrm{B}}$ and $\mathcal{S}_3^{\mathrm{B}}$ satisfy

$$x_2 + x_3 = n_2 = 2,$$

the call blocking for classes 2 and 3 would have been determined using Erlang's formula (L7/20) with parameters $n = n_2 = 2$ ja $a = a_2 + a_3 = 1 + 2 = 3$ erl:

$$B_2 = B_3 = \mathrm{Erl}(n_2, a_2 + a_3) = \mathrm{Erl}(2,3) = \frac{\frac{9}{2}}{1+3+\frac{9}{2}} = \frac{9}{17} = 0.53$$

(b) The Product Bound method considers first blocking probability $B(j)$in each link j separately (as if the other links were of infinite capacity). Link 1 is used by classes 1 and 3 so that (L10/19)

$$B(1) = \mathrm{Erl}(n_1, a_1 + a_3) = \mathrm{Erl}(2,2) = \frac{2}{1+2+2} = \frac{2}{5} = 0.40,$$

while link 2 is used by classes 2 and 3 so that (L10/19)

$$B(2) = \text{Erl}(n_2, a_2 + a_3) = \text{Erl}(2, 3) = \frac{\frac{9}{2}}{1 + 3 + \frac{9}{2}} = \frac{9}{17} = 0.53$$

These linkwise blocking probabilities lead to the following end-to-end blocking probability approximations for each class r (L10/20):

$$\begin{aligned} B_1 &\approx 1 - (1 - B(1)) = B(1) = \frac{2}{5} = 0.40, \\ B_2 &\approx 1 - (1 - B(2)) = B(2) = \frac{9}{17} = 0.53, \\ B_3 &\approx 1 - (1 - B(1))(1 - B(2)) = B(1) + B(2) - B(1)B(2) = \frac{61}{85} = 0.72 \end{aligned}$$

Note that these approximate values differ from the exact values for classes 1 and 3, but for class 2 the two values are equal. Explain why.

D10/3 The system consists of six links with equal capacities $C_j = 1$ Gbps. The topology and the routes of this network are shown in Figure 6.

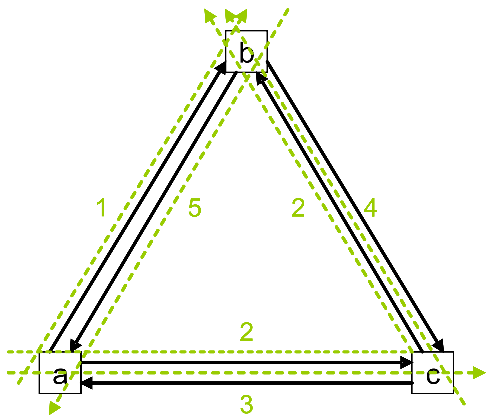

Figure 6: [D10/3] Topology and routes.

(a) The packet arrival rates λ_j (in packets/ms) for different links (L10/32) are

$$\begin{aligned} \lambda_{\text{ab}} &= \lambda(1) = 40, \\ \lambda_{\text{ba}} &= \lambda(5) = 20, \\ \lambda_{\text{bc}} &= 0, \\ \lambda_{\text{cb}} &= \lambda(2) + \lambda(4) = 80, \\ \lambda_{\text{ca}} &= 0, \\ \lambda_{\text{ac}} &= \lambda(2) + \lambda(3) = 80 \end{aligned}$$

Since the mean packet length is $L = 1250 \cdot 8 = 10^4$ b and the link capacity equals $C = 10^6$ b/ms for all links, the service rate (in packets/ms) for any link becomes

$$\mu_j = \frac{C}{L} = \frac{10^6}{10^4} = 100$$

Thus, the link capacities $\rho_j = \lambda_j/\mu_j$ are as follows:

$$\begin{aligned}
\rho_{\text{ab}} &= 0.40, \\
\rho_{\text{ba}} &= 0.20, \\
\rho_{\text{bc}} &= 0, \\
\rho_{\text{cb}} &= 0.80, \\
\rho_{\text{ca}} &= 0, \\
\rho_{\text{ac}} &= 0.80
\end{aligned}$$

(b) The mean packet delays for different links are determined using the M/M/1 model (L10/35),

$$\bar{T}_j = \frac{1}{\mu_j - \lambda_j},$$

which results in the following mean link delays (in ms):

$$\begin{aligned}
\bar{T}_{\text{ab}} &= \frac{1}{100-40} = \frac{1}{60} = 0.017, \\
\bar{T}_{\text{ba}} &= \frac{1}{100-20} = \frac{1}{80} = 0.013, \\
\bar{T}_{\text{bc}} &= \frac{1}{100-0} = \frac{1}{100} = 0.001, \\
\bar{T}_{\text{cb}} &= \frac{1}{100-80} = \frac{1}{20} = 0.050, \\
\bar{T}_{\text{ca}} &= \frac{1}{100-0} = \frac{1}{100} = 0.001, \\
\bar{T}_{\text{ac}} &= \frac{1}{100-80} = \frac{1}{20} = 0.050
\end{aligned}$$

The mean end-to-end delay $\bar{T}(r)$ of a packet following route r is now the sum of corresponding link delays:

$$\begin{aligned}
\bar{T}(1) &= \bar{T}_{\text{ab}} = \frac{1}{60} = 0.017, \\
\bar{T}(2) &= \bar{T}_{\text{ac}} + \bar{T}_{\text{cb}} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} = 0.100, \\
\bar{T}(3) &= \bar{T}_{\text{ac}} = \frac{1}{20} = 0.050, \\
\bar{T}(4) &= \bar{T}_{\text{cb}} = \frac{1}{20} = 0.050, \\
\bar{T}(5) &= \bar{T}_{\text{ba}} = \frac{1}{80} = 0.013
\end{aligned}$$